

# A Story of Ratios<sup>®</sup>

## Eureka Math<sup>™</sup>

### Grade 8, Module 4

#### Student File\_A

*Contains copy-ready classwork and homework  
as well as templates (including cut outs)*

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10 9 8 7 6 5 4 3 2 1

## Lesson 1: Writing Equations Using Symbols

### Classwork

#### Exercises

Write each of the following statements using symbolic language.

1. The sum of four consecutive even integers is  $-28$ .
2. A number is four times larger than the square of half the number.
3. Steven has some money. If he spends \$9.00, then he will have  $\frac{3}{5}$  of the amount he started with.
4. The sum of a number squared and three less than twice the number is 129.
5. Miriam read a book with an unknown number of pages. The first week, she read five less than  $\frac{1}{3}$  of the pages. The second week, she read 171 more pages and finished the book. Write an equation that represents the total number of pages in the book.

**Lesson Summary**

Begin all word problems by defining your variables. State clearly what you want each symbol to represent.

Written mathematical statements can be represented as more than one correct symbolic statement.

Break complicated problems into smaller parts, or try working them with simpler numbers.

**Problem Set**

Write each of the following statements using symbolic language.

1. Bruce bought two books. One book costs \$4.00 more than three times the other. Together, the two books cost him \$72.
2. Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is 55 years.
3. The sum of three consecutive integers is 1,623.
4. One number is six more than another number. The sum of their squares is 90.
5. When you add 18 to  $\frac{1}{4}$  of a number, you get the number itself.
6. When a fraction of 17 is taken away from 17, what remains exceeds one-third of seventeen by six.
7. The sum of two consecutive even integers divided by four is 189.5.
8. Subtract seven more than twice a number from the square of one-third of the number to get zero.
9. The sum of three consecutive integers is 42. Let  $x$  be the middle of the three integers. Transcribe the statement accordingly.

## Lesson 2: Linear and Nonlinear Expressions in $x$

### Classwork

#### Exercises

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. The sum of a number and four times the number
2. The product of five and a number
3. Multiply six and the reciprocal of the quotient of a number and seven.
4. Twice a number subtracted from four times a number, added to 15
5. The square of the sum of six and a number
6. The cube of a positive number divided by the square of the same positive number

7. The sum of four consecutive numbers
  
  
  
  
  
  
  
  
  
  
8. Four subtracted from the reciprocal of a number
  
  
  
  
  
  
  
  
  
  
9. Half of the product of a number multiplied by itself three times
  
  
  
  
  
  
  
  
  
  
10. The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and  $\frac{2}{3}$  of the remaining pages today
  
  
  
  
  
  
  
  
  
  
11. An admission fee of \$10 plus an additional \$2 per game
  
  
  
  
  
  
  
  
  
  
12. Five more than four times a number and then twice that sum

**Lesson Summary**

A *linear expression* is an expression that is equivalent to the sum or difference of one or more expressions where each expression is either a number, a variable, or a product of a number and a variable.

A linear expression in  $x$  can be represented by terms whose variable  $x$  is raised to either a power of 0 or 1. For example,  $4 + 3x$ ,  $7x + x - 15$ , and  $\frac{1}{2}x + 7 - 2$  are all linear expressions in  $x$ . A nonlinear expression in  $x$  has terms where  $x$  is raised to a power that is not 0 or 1. For example,  $2x^2 - 9$ ,  $-6x^{-3} + 8 + x$ , and  $\frac{1}{x} + 8$  are all nonlinear expressions in  $x$ .

**Problem Set**

Write each of the following statements as a mathematical expression. State whether the expression is linear or nonlinear. If it is nonlinear, then explain why.

1. A number decreased by three squared
2. The quotient of two and a number, subtracted from seventeen
3. The sum of thirteen and twice a number
4. 5.2 more than the product of seven and a number
5. The sum that represents the number of tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday
6. The product of 19 and a number, subtracted from the reciprocal of the number cubed
7. The product of 15 and a number, and then the product multiplied by itself four times
8. A number increased by five and then divided by two
9. Eight times the result of subtracting three from a number
10. The sum of twice a number and four times a number subtracted from the number squared
11. One-third of the result of three times a number that is increased by 12

12. Five times the sum of one-half and a number
13. Three-fourths of a number multiplied by seven
14. The sum of a number and negative three, multiplied by the number
15. The square of the difference between a number and 10



4. Lisa solved the equation  $x + 6 = 8 + 7x$  and claimed that the solution is  $x = -\frac{1}{3}$ . Is she correct? Explain.
5. Angel transformed the following equation from  $6x + 4 - x = 2(x + 1)$  to  $10 = 2(x + 1)$ . He then stated that the solution to the equation is  $x = 4$ . Is he correct? Explain.
6. Claire was able to verify that  $x = 3$  was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of  $x = 3$ .
7. Does an equation always have a solution? Could you come up with an equation that does not have a solution?

**Lesson Summary**

An equation is a statement about equality between two expressions. If the expression on the left side of the equal sign has the same value as the expression on the right side of the equal sign, then you have a true equation.

A solution of a linear equation in  $x$  is a number, such that when all instances of  $x$  are replaced with the number, the left side will equal the right side. For example, 2 is a solution to  $3x + 4 = x + 8$  because when  $x = 2$ , the left side of the equation is

$$\begin{aligned}3x + 4 &= 3(2) + 4 \\ &= 6 + 4 \\ &= 10,\end{aligned}$$

and the right side of the equation is

$$\begin{aligned}x + 8 &= 2 + 8 \\ &= 10.\end{aligned}$$

Since  $10 = 10$ , then  $x = 2$  is a solution to the linear equation  $3x + 4 = x + 8$ .

**Problem Set**

- Given that  $2x + 7 = 27$  and  $3x + 1 = 28$ , does  $2x + 7 = 3x + 1$ ? Explain.
- Is  $-5$  a solution to the equation  $6x + 5 = 5x + 8 + 2x$ ? Explain.
- Does  $x = 1.6$  satisfy the equation  $6 - 4x = -\frac{x}{4}$ ? Explain.
- Use the linear equation  $3(x + 1) = 3x + 3$  to answer parts (a)–(d).
  - Does  $x = 5$  satisfy the equation above? Explain.
  - Is  $x = -8$  a solution of the equation above? Explain.
  - Is  $x = \frac{1}{2}$  a solution of the equation above? Explain.
  - What interesting fact about the equation  $3(x + 1) = 3x + 3$  is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?



3. Solve the linear equation  $x - 9 = \frac{3}{5}x$ . State the property that justifies your first step and why you chose it.

4. Solve the linear equation  $29 - 3x = 5x + 5$ . State the property that justifies your first step and why you chose it.

5. Solve the linear equation  $\frac{1}{3}x - 5 + 171 = x$ . State the property that justifies your first step and why you chose it.

**Lesson Summary**

The properties of equality, shown below, are used to transform equations into simpler forms. If  $A, B, C$  are rational numbers, then:

- If  $A = B$ , then  $A + C = B + C$ . Addition property of equality
- If  $A = B$ , then  $A - C = B - C$ . Subtraction property of equality
- If  $A = B$ , then  $A \cdot C = B \cdot C$ . Multiplication property of equality
- If  $A = B$ , then  $\frac{A}{C} = \frac{B}{C}$ , where  $C$  is not equal to zero. Division property of equality

To solve an equation, transform the equation until you get to the form of  $x$  equal to a constant ( $x = 5$ , for example).

**Problem Set**

For each problem, show your work, and check that your solution is correct.

1. Solve the linear equation  $x + 4 + 3x = 72$ . State the property that justifies your first step and why you chose it.
2. Solve the linear equation  $x + 3 + x - 8 + x = 55$ . State the property that justifies your first step and why you chose it.
3. Solve the linear equation  $\frac{1}{2}x + 10 = \frac{1}{4}x + 54$ . State the property that justifies your first step and why you chose it.
4. Solve the linear equation  $\frac{1}{4}x + 18 = x$ . State the property that justifies your first step and why you chose it.
5. Solve the linear equation  $17 - x = \frac{1}{3} \cdot 15 + 6$ . State the property that justifies your first step and why you chose it.
6. Solve the linear equation  $\frac{x+x+2}{4} = 189.5$ . State the property that justifies your first step and why you chose it.

7. Alysha solved the linear equation  $2x - 3 - 8x = 14 + 2x - 1$ . Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.

$$2x - 3 - 8x = 14 + 2x - 1$$

$$-6x - 3 = 13 + 2x$$

$$-6x - 3 + 3 = 13 + 3 + 2x$$

$$-6x = 16 + 2x$$

$$-6x + 2x = 16$$

$$-4x = 16$$

$$\frac{-4}{-4}x = \frac{16}{-4}$$

$$x = -4$$

## Lesson 5: Writing and Solving Linear Equations

### Classwork

#### Example 1

One angle is five degrees less than three times the degree measure of another angle. Together, the angles measures have a sum of  $143^\circ$ . What is the measure of each angle?

#### Example 2

Given a right triangle, find the degree measure of the angles if one angle is ten degrees more than four times the degree measure of the other angle and the third angle is the right angle.



4. One angle measures nine more than six times a number. A sequence of rigid motions maps the angle onto another angle that is described as being thirty less than nine times the number. What is the measure of the angle in degrees?
5. A right triangle is described as having an angle of measure six less than negative two times a number, another angle measure that is three less than negative one-fourth the number, and a right angle. What are the measures of the angles in degrees?
6. One angle is one less than six times the measure of another. The two angles are complementary angles. Find the measure of each angle in degrees.

**Problem Set**

For each of the following problems, write an equation and solve.

1. The measure of one angle is thirteen less than five times the measure of another angle. The sum of the measures of the two angles is  $140^\circ$ . Determine the measure of each angle in degrees.
2. An angle measures seventeen more than three times a number. Its supplement is three more than seven times the number. What is the measure of each angle in degrees?
3. The angles of a triangle are described as follows:  $\angle A$  is the largest angle; its measure is twice the measure of  $\angle B$ . The measure of  $\angle C$  is 2 less than half the measure of  $\angle B$ . Find the measures of the three angles in degrees.
4. A pair of corresponding angles are described as follows: The measure of one angle is five less than seven times a number, and the measure of the other angle is eight more than seven times the number. Are the angles congruent? Why or why not?
5. The measure of one angle is eleven more than four times a number. Another angle is twice the first angle's measure. The sum of the measures of the angles is  $195^\circ$ . What is the measure of each angle in degrees?
6. Three angles are described as follows:  $\angle B$  is half the size of  $\angle A$ . The measure of  $\angle C$  is equal to one less than two times the measure of  $\angle B$ . The sum of  $\angle A$  and  $\angle B$  is  $114^\circ$ . Can the three angles form a triangle? Why or why not?

## Lesson 6: Solutions of a Linear Equation

### Classwork

#### Exercises

Find the value of  $x$  that makes the equation true.

1.  $17 - 5(2x - 9) = -(-6x + 10) + 4$

2.  $-(x - 7) + \frac{5}{3} = 2(x + 9)$

3.  $\frac{4}{9} + 4(x - 1) = \frac{28}{9} - (x - 7x) + 1$

4.  $5(3x + 4) - 2x = 7x - 3(-2x + 11)$

5.  $7x - (3x + 5) - 8 = \frac{1}{2}(8x + 20) - 7x + 5$

6. Write at least three equations that have no solution.

**Lesson Summary**

The distributive property is used to expand expressions. For example, the expression  $2(3x - 10)$  is rewritten as  $6x - 20$  after the distributive property is applied.

The distributive property is used to simplify expressions. For example, the expression  $7x + 11x$  is rewritten as  $(7 + 11)x$  and  $18x$  after the distributive property is applied.

The distributive property is applied only to terms within a group:

$$4(3x + 5) - 2 = 12x + 20 - 2.$$

Notice that the term  $-2$  is not part of the group and, therefore, not multiplied by 4.

When an equation is transformed into an untrue sentence, such as  $5 \neq 11$ , we say the equation has *no solution*.

**Problem Set**

Transform the equation if necessary, and then solve it to find the value of  $x$  that makes the equation true.

1.  $x - (9x - 10) + 11 = 12x + 3\left(-2x + \frac{1}{3}\right)$

2.  $7x + 8\left(x + \frac{1}{4}\right) = 3(6x - 9) - 8$

3.  $-4x - 2(8x + 1) = -(-2x - 10)$

4.  $11(x + 10) = 132$

5.  $37x + \frac{1}{2} - \left(x + \frac{1}{4}\right) = 9(4x - 7) + 5$

6.  $3(2x - 14) + x = 15 - (-9x - 5)$

7.  $8(2x + 9) = 56$

## Lesson 7: Classification of Solutions

### Classwork

#### Exercises

Solve each of the following equations for  $x$ .

1.  $7x - 3 = 5x + 5$

2.  $7x - 3 = 7x + 5$

3.  $7x - 3 = -3 + 7x$

Give a brief explanation as to what kind of solution(s) you expect the following linear equations to have. Transform the equations into a simpler form if necessary.

4.  $11x - 2x + 15 = 8 + 7 + 9x$

5.  $3(x - 14) + 1 = -4x + 5$

6.  $-3x + 32 - 7x = -2(5x + 10)$

7.  $\frac{1}{2}(8x + 26) = 13 + 4x$

8. Write two equations that have no solutions.

9. Write two equations that have one unique solution each.

10. Write two equations that have infinitely many solutions.

**Lesson Summary**

There are three classifications of solutions to linear equations: one solution (unique solution), no solution, or infinitely many solutions.

Equations with no solution will, after being simplified, have coefficients of  $x$  that are the same on both sides of the equal sign and constants that are different. For example,  $x + b = x + c$ , where  $b$  and  $c$  are constants that are not equal. A numeric example is  $8x + 5 = 8x - 3$ .

Equations with infinitely many solutions will, after being simplified, have coefficients of  $x$  and constants that are the same on both sides of the equal sign. For example,  $x + a = x + a$ , where  $a$  is a constant. A numeric example is  $6x + 1 = 1 + 6x$ .

**Problem Set**

1. Give a brief explanation as to what kind of solution(s) you expect for the linear equation  $18x + \frac{1}{2} = 6(3x + 25)$ . Transform the equation into a simpler form if necessary.
2. Give a brief explanation as to what kind of solution(s) you expect for the linear equation  $8 - 9x = 15x + 7 + 3x$ . Transform the equation into a simpler form if necessary.
3. Give a brief explanation as to what kind of solution(s) you expect for the linear equation  $5(x + 9) = 5x + 45$ . Transform the equation into a simpler form if necessary.
4. Give three examples of equations where the solution will be unique; that is, only one solution is possible.
5. Solve one of the equations you wrote in Problem 4, and explain why it is the only solution.
6. Give three examples of equations where there will be no solution.
7. Attempt to solve one of the equations you wrote in Problem 6, and explain why it has no solution.
8. Give three examples of equations where there will be infinitely many solutions.
9. Attempt to solve one of the equations you wrote in Problem 8, and explain why it has infinitely many solutions.

## Lesson 8: Linear Equations in Disguise

### Classwork

#### Example 3

Can this equation be solved?

$$\frac{6+x}{7x+\frac{2}{3}} = \frac{3}{8}$$

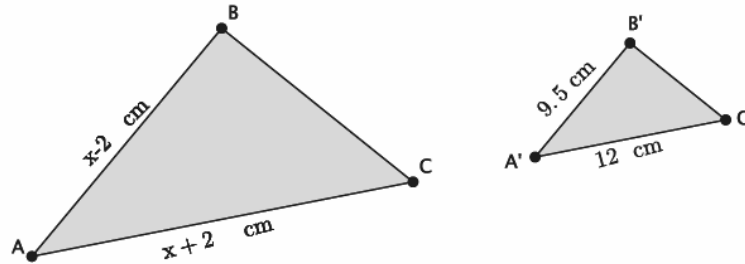
#### Example 4

Can this equation be solved?

$$\frac{7}{3x+9} = \frac{1}{8}$$

**Example 5**

In the diagram below,  $\triangle ABC \sim \triangle A'B'C'$ . Using what we know about similar triangles, we can determine the value of  $x$ .

**Exercises**

Solve the following equations of rational expressions, if possible.

1.  $\frac{2x + 1}{9} = \frac{1 - x}{6}$

$$2. \frac{5 + 2x}{3x - 1} = \frac{6}{7}$$

$$3. \frac{x + 9}{12} = \frac{-2x - \frac{1}{2}}{3}$$

$$4. \frac{8}{3 - 4x} = \frac{5}{2x + \frac{1}{4}}$$

### Lesson Summary

Some proportions are linear equations in disguise and are solved the same way we normally solve proportions.

When multiplying a fraction with more than one term in the numerator and/or denominator by a number, put the expressions with more than one term in parentheses so that you remember to use the distributive property when transforming the equation. For example:

$$\frac{x + 4}{2x - 5} = \frac{3}{5}$$

$$5(x + 4) = 3(2x - 5).$$

The equation  $5(x + 4) = 3(2x - 5)$  is now clearly a linear equation and can be solved using the properties of equality.

### Problem Set

Solve the following equations of rational expressions, if possible. If an equation cannot be solved, explain why.

1.  $\frac{5}{6x - 2} = \frac{-1}{x + 1}$

6.  $\frac{2x + 5}{2} = \frac{3x - 2}{6}$

2.  $\frac{4 - x}{8} = \frac{7x - 1}{3}$

7.  $\frac{6x + 1}{3} = \frac{9 - x}{7}$

3.  $\frac{3x}{x + 2} = \frac{5}{9}$

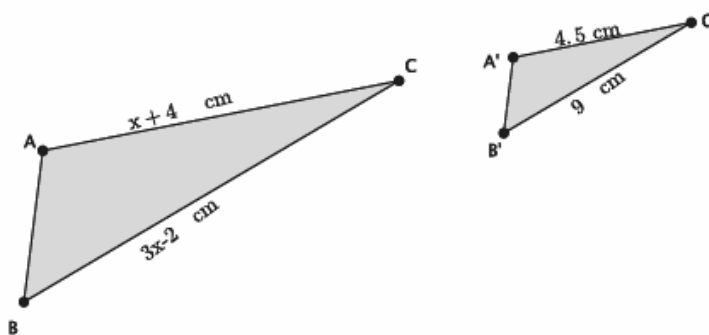
8.  $\frac{\frac{1}{3}x - 8}{12} = \frac{-2 - x}{15}$

4.  $\frac{\frac{1}{2}x + 6}{3} = \frac{x - 3}{2}$

9.  $\frac{3 - x}{1 - x} = \frac{3}{2}$

5.  $\frac{7 - 2x}{6} = \frac{x - 5}{1}$

10. In the diagram below,  $\triangle ABC \sim \triangle A'B'C'$ . Determine the lengths of  $\overline{AC}$  and  $\overline{BC}$ .



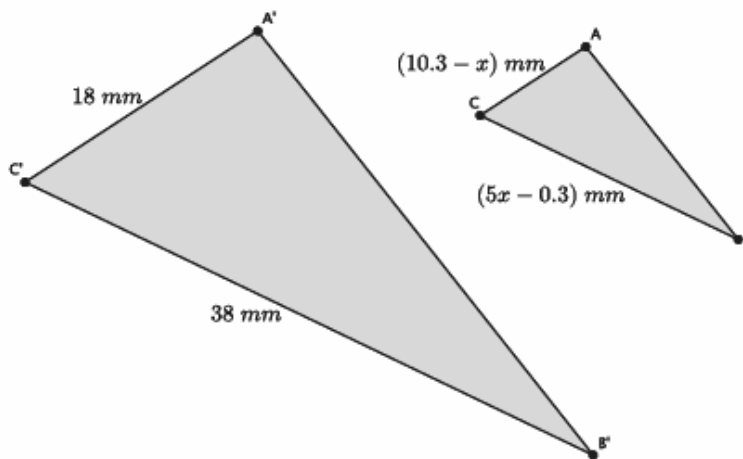


3. Marvin paid an entrance fee of \$5 plus an additional \$1.25 per game at a local arcade. Altogether, he spent \$26.25. Write and solve an equation to determine how many games Marvin played.
4. The sum of four consecutive integers is  $-26$ . What are the integers?
5. A book has  $x$  pages. How many pages are in the book if Maria read 45 pages of a book on Monday,  $\frac{1}{2}$  the book on Tuesday, and the remaining 72 pages on Wednesday?
6. A number increased by 5 and divided by 2 is equal to 75. What is the number?

7. The sum of thirteen and twice a number is seven less than six times a number. What is the number?
8. The width of a rectangle is 7 less than twice the length. If the perimeter of the rectangle is 43.6 inches, what is the area of the rectangle?
9. Two hundred and fifty tickets for the school dance were sold. On Monday, 35 tickets were sold. An equal number of tickets were sold each day for the next five days. How many tickets were sold on one of those days?

10. Shonna skateboarded for some number of minutes on Monday. On Tuesday, she skateboarded for twice as many minutes as she did on Monday, and on Wednesday, she skateboarded for half the sum of minutes from Monday and Tuesday. Altogether, she skateboarded for a total of three hours. How many minutes did she skateboard each day?

11. In the diagram below,  $\triangle ABC \sim \triangle A'B'C'$ . Determine the length of  $\overline{AC}$  and  $\overline{BC}$ .



## Problem Set

1. You forward an e-card that you found online to three of your friends. They liked it so much that they forwarded it on to four of their friends, who then forwarded it on to four of their friends, and so on. The number of people who saw the e-card is shown below. Let  $S_1$  represent the number of people who saw the e-card after one step, let  $S_2$  represent the number of people who saw the e-card after two steps, and so on.

$$S_1 = 3$$

$$S_2 = 3 + 3 \cdot 4$$

$$S_3 = 3 + 3 \cdot 4 + 3 \cdot 4^2$$

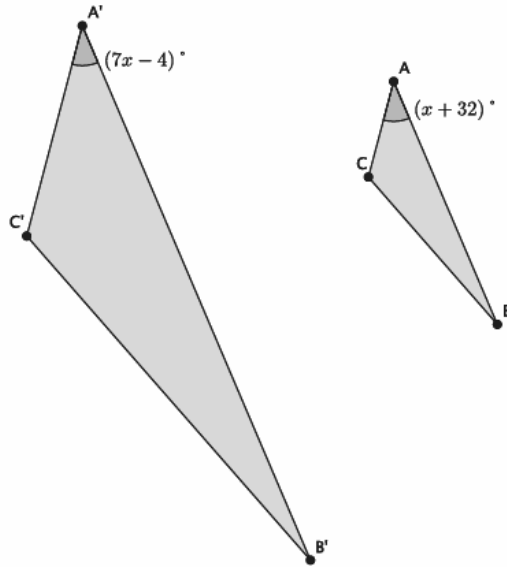
$$S_4 = 3 + 3 \cdot 4 + 3 \cdot 4^2 + 3 \cdot 4^3$$

- Find the pattern in the equations.
- Assuming the trend continues, how many people will have seen the e-card after 10 steps?
- How many people will have seen the e-card after  $n$  steps?

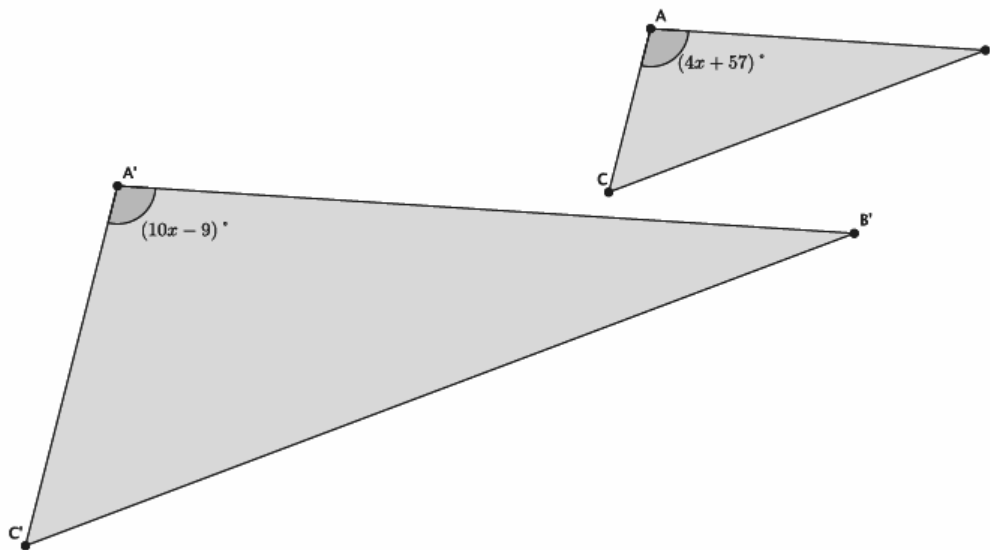
For each of the following questions, write an equation, and solve to find each answer.

- Lisa has a certain amount of money. She spent \$39 and has  $\frac{3}{4}$  of the original amount left. How much money did she have originally?
- The length of a rectangle is 4 more than 3 times the width. If the perimeter of the rectangle is 18.4 cm, what is the area of the rectangle?
- Eight times the result of subtracting 3 from a number is equal to the number increased by 25. What is the number?
- Three consecutive odd integers have a sum of 3. What are the numbers?
- Each month, Liz pays \$35 to her phone company just to use the phone. Each text she sends costs her an additional \$0.05. In March, her phone bill was \$72.60. In April, her phone bill was \$65.85. How many texts did she send each month?
- Claudia is reading a book that has 360 pages. She read some of the book last week. She plans to read 46 pages today. When she does, she will be  $\frac{4}{5}$  of the way through the book. How many pages did she read last week?

8. In the diagram below,  $\triangle ABC \sim \triangle A'B'C'$ . Determine the measure of  $\angle A$ .



9. In the diagram below,  $\triangle ABC \sim \triangle A'B'C'$ . Determine the measure of  $\angle A$ .



## Lesson 10: A Critical Look at Proportional Relationships

### Classwork

#### Example 1

Paul walks 2 miles in 25 minutes. How many miles can Paul walk in 137.5 minutes?

Time (in minutes)	Distance (in miles)
25	2

**Exercises**

- Wesley walks at a constant speed from his house to school 1.5 miles away. It took him 25 minutes to get to school.
  - What fraction represents his constant speed,  $C$ ?
  - You want to know how many miles he has walked after 15 minutes. Let  $y$  represent the distance he traveled after 15 minutes of walking at the given constant speed. Write a fraction that represents the constant speed,  $C$ , in terms of  $y$ .
  - Write the fractions from parts (a) and (b) as a proportion, and solve to find how many miles Wesley walked after 15 minutes.
  - Let  $y$  be the distance in miles that Wesley traveled after  $x$  minutes. Write a linear equation in two variables that represents how many miles Wesley walked after  $x$  minutes.
- Stefanie drove at a constant speed from her apartment to her friend's house 20 miles away. It took her 45 minutes to reach her destination.
  - What fraction represents her constant speed,  $C$ ?

- b. What fraction represents constant speed,  $C$ , if it takes her  $x$  number of minutes to get halfway to her friend's house?
- c. Write and solve a proportion using the fractions from parts (a) and (b) to determine how many minutes it takes her to get to the halfway point.
- d. Write a two-variable equation to represent how many miles Stefanie can drive over any time interval.
3. The equation that represents how many miles,  $y$ , Dave travels after  $x$  hours is  $y = 50x + 15$ . Use the equation to complete the table below.

$x$ (hours)	Linear Equation: $y = 50x + 15$	$y$ (miles)
1	$y = 50(1) + 15$	65
2		
3		
3.5		
4.1		

**Lesson Summary**

Average speed is found by taking the total distance traveled in a given time interval, divided by the time interval.

If  $y$  is the total distance traveled in a given time interval  $x$ , then  $\frac{y}{x}$  is the average speed.

If we assume the same average speed over any time interval, then we have constant speed, which can then be used to express a linear equation in two variables relating distance and time.

If  $\frac{y}{x} = C$ , where  $C$  is a constant, then you have constant speed.

**Problem Set**

- Eman walks from the store to her friend's house, 2 miles away. It takes her 35 minutes.
  - What fraction represents her constant speed,  $C$ ?
  - Write the fraction that represents her constant speed,  $C$ , if she walks  $y$  miles in 10 minutes.
  - Write and solve a proportion using the fractions from parts (a) and (b) to determine how many miles she walks after 10 minutes. Round your answer to the hundredths place.
  - Write a two-variable equation to represent how many miles Eman can walk over any time interval.
- Erika drives from school to soccer practice 1.3 miles away. It takes her 7 minutes.
  - What fraction represents her constant speed,  $C$ ?
  - What fraction represents her constant speed,  $C$ , if it takes her  $x$  minutes to drive exactly 1 mile?
  - Write and solve a proportion using the fractions from parts (a) and (b) to determine how much time it takes her to drive exactly 1 mile. Round your answer to the tenths place.
  - Write a two-variable equation to represent how many miles Erika can drive over any time interval.
- Darla drives at a constant speed of 45 miles per hour.
  - If she drives for  $y$  miles and it takes her  $x$  hours, write the two-variable equation to represent the number of miles Darla can drive in  $x$  hours.
  - Darla plans to drive to the market 14 miles from her house, then to the post office 3 miles from the market, and then return home, which is 15 miles from the post office. Assuming she drives at a constant speed the entire time, how long will it take her to run her errands and get back home? Round your answer to the hundredths place.
- Aaron walks from his sister's house to his cousin's house, a distance of 4 miles, in 80 minutes. How far does he walk in 30 minutes?

5. Carlos walks 4 miles every night for exercise. It takes him exactly 63 minutes to finish his walk.
- Assuming he walks at a constant rate, write an equation that represents how many miles,  $y$ , Carlos can walk in  $x$  minutes.
  - Use your equation from part (a) to complete the table below. Use a calculator, and round all values to the hundredths place.

$x$ (minutes)	Linear Equation:	$y$ (miles)
15		
30		
40		
60		
75		

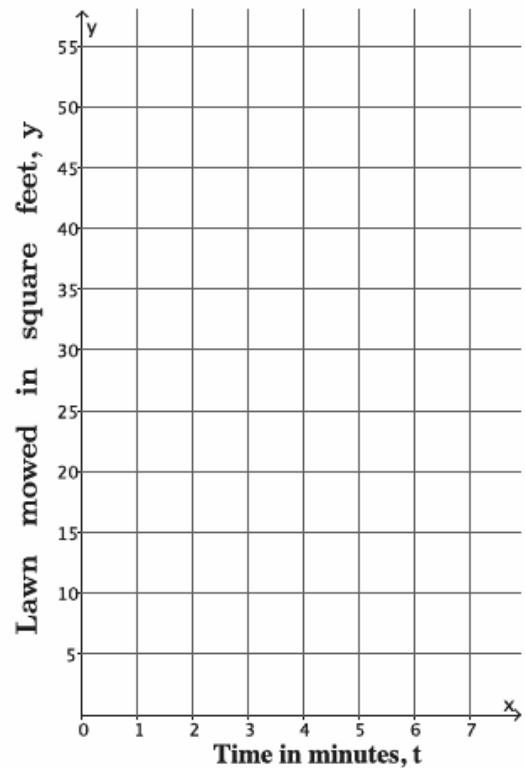
## Lesson 11: Constant Rate

### Classwork

#### Example 1

Pauline mows a lawn at a constant rate. Suppose she mows a 35-square-foot lawn in 2.5 minutes. What area, in square feet, can she mow in 10 minutes?  $t$  minutes?

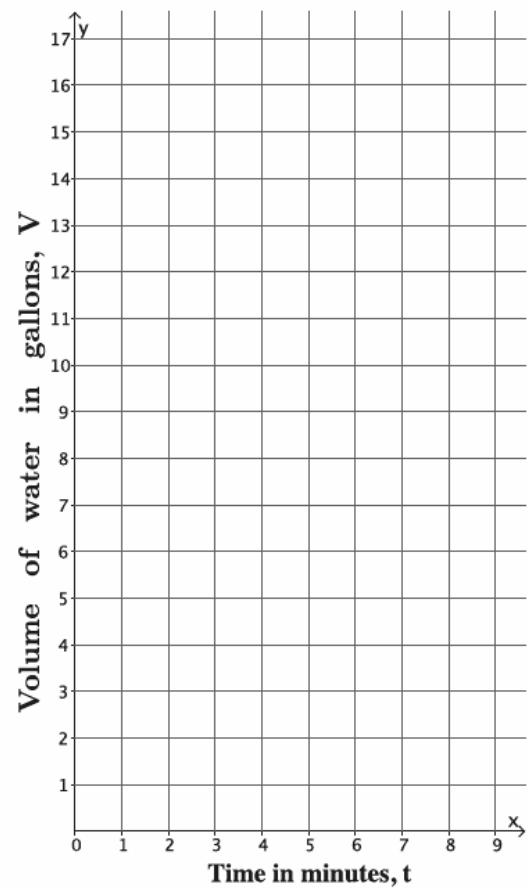
$t$ (time in minutes)	Linear Equation:	$y$ (area in square feet)



## Example 2

Water flows at a constant rate out of a faucet. Suppose the volume of water that comes out in three minutes is 10.5 gallons. How many gallons of water come out of the faucet in  $t$  minutes?

$t$ (time in minutes)	Linear Equation:	$V$ (in gallons)
0		
1		
2		
3		
4		

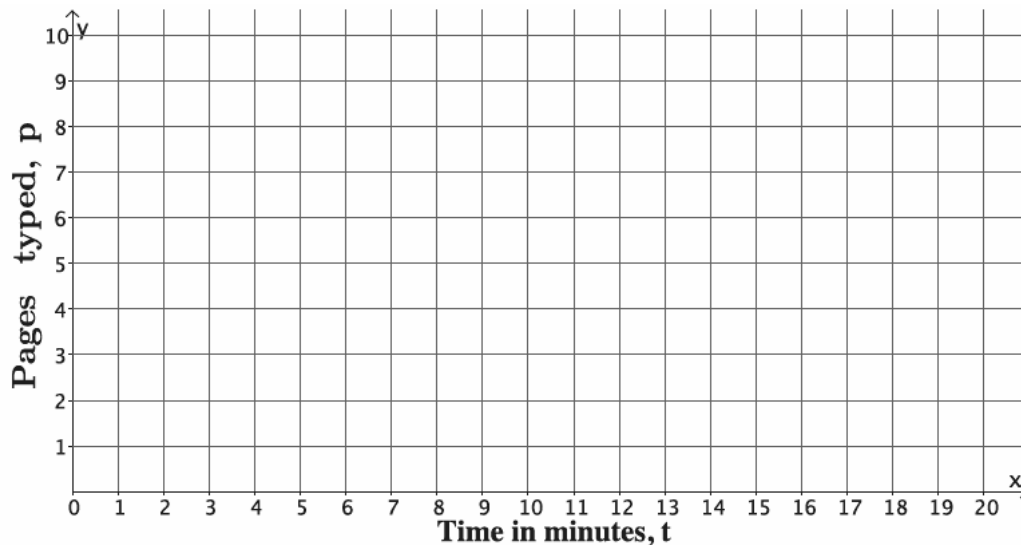


## Exercises

1. Juan types at a constant rate. He can type a full page of text in  $3\frac{1}{2}$  minutes. We want to know how many pages,  $p$ , Juan can type after  $t$  minutes.
- Write the linear equation in two variables that represents the number of pages Juan types in any given time interval.
  - Complete the table below. Use a calculator, and round your answers to the tenths place.

$t$ (time in minutes)	Linear Equation:	$p$ (pages typed)
0		
5		
10		
15		
20		

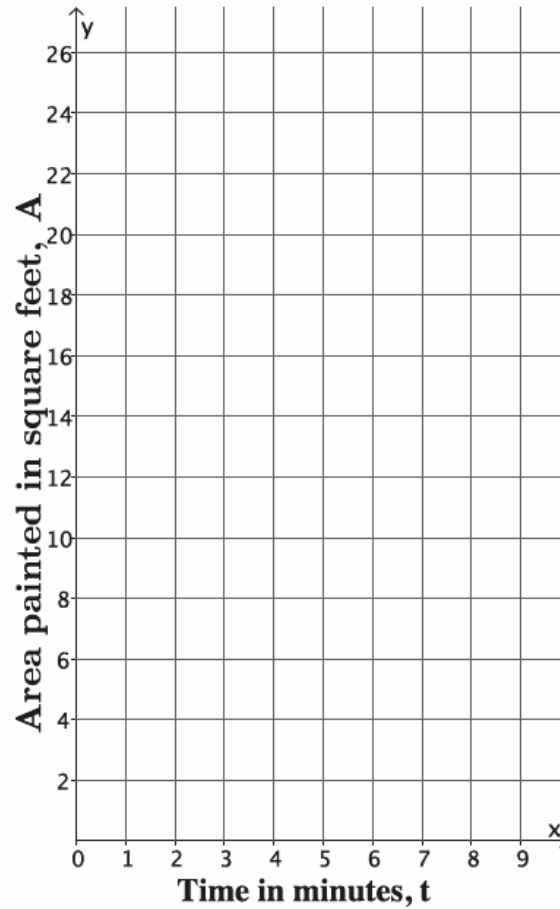
- Graph the data on a coordinate plane.



- d. About how long would it take Juan to type a 5-page paper? Explain.
2. Emily paints at a constant rate. She can paint 32 square feet in 5 minutes. What area,  $A$ , in square feet, can she paint in  $t$  minutes?
- a. Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.
- b. Complete the table below. Use a calculator, and round answers to the tenths place.

$t$ (time in minutes)	Linear Equation:	$A$ (area painted in square feet)
0		
1		
2		
3		
4		

- c. Graph the data on a coordinate plane.



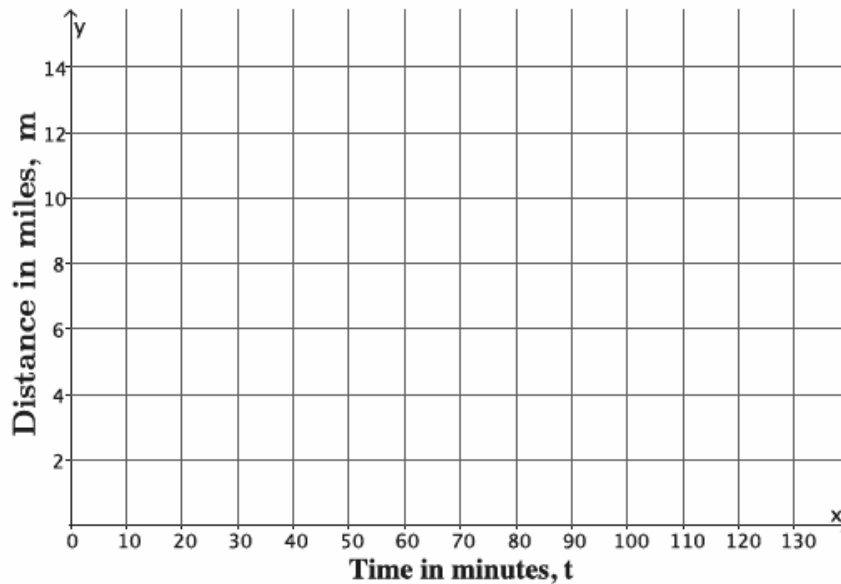
- d. About how many square feet can Emily paint in  $2\frac{1}{2}$  minutes? Explain.

3. Joseph walks at a constant speed. He walked to a store that is one-half mile away in 6 minutes. How many miles,  $m$ , can he walk in  $t$  minutes?
- a. Write the linear equation in two variables that represents the number of miles Joseph can walk in any given time interval,  $t$ .

- b. Complete the table below. Use a calculator, and round answers to the tenths place.

$t$ (time in minutes)	Linear Equation:	$m$ (distance in miles)
0		
30		
60		
90		
120		

- c. Graph the data on a coordinate plane.



- d. Joseph's friend lives 4 miles away from him. About how long would it take Joseph to walk to his friend's house? Explain.

**Lesson Summary**

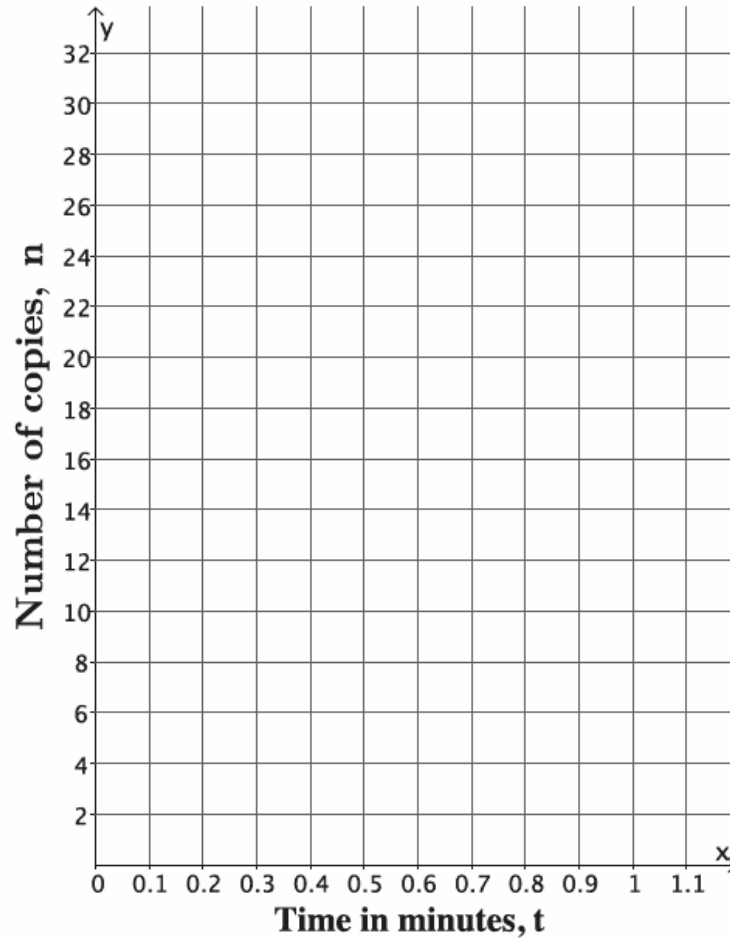
When constant rate is stated for a given problem, then you can express the situation as a two-variable equation. The equation can be used to complete a table of values that can then be graphed on a coordinate plane.

**Problem Set**

- A train travels at a constant rate of 45 miles per hour.
  - What is the distance,  $d$ , in miles, that the train travels in  $t$  hours?
  - How many miles will it travel in 2.5 hours?
- Water is leaking from a faucet at a constant rate of  $\frac{1}{3}$  gallon per minute.
  - What is the amount of water,  $w$ , in gallons per minute, that is leaked from the faucet after  $t$  minutes?
  - How much water is leaked after an hour?
- A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.
  - How many cars,  $y$ , can be assembled in  $t$  hours?
  - How many cars can be assembled in a week?
- A copy machine makes copies at a constant rate. The machine can make 80 copies in  $2\frac{1}{2}$  minutes.
  - Write an equation to represent the number of copies,  $n$ , that can be made over any time interval in minutes,  $t$ .
  - Complete the table below.

$t$ (time in minutes)	Linear Equation:	$n$ (number of copies)
0		
0.25		
0.5		
0.75		
1		

- c. Graph the data on a coordinate plane.

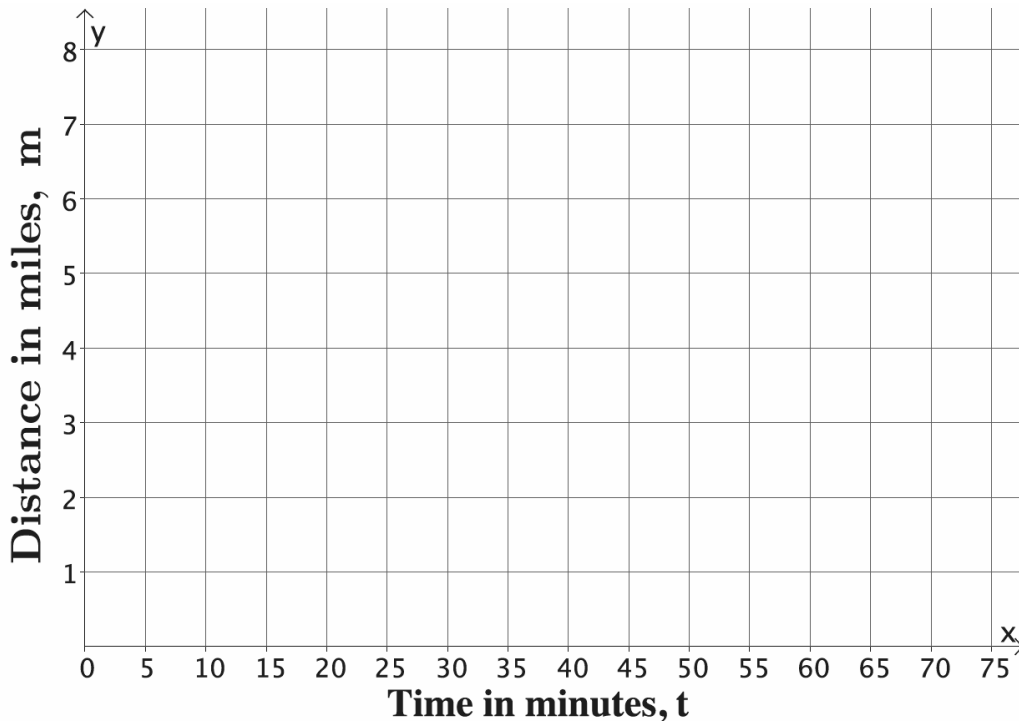


- d. The copy machine runs for 20 seconds and then jams. About how many copies were made before the jam occurred? Explain.

5. Connor runs at a constant rate. It takes him 34 minutes to run 4 miles.
- Write the linear equation in two variables that represents the number of miles Connor can run in any given time interval in minutes,  $t$ .
  - Complete the table below. Use a calculator, and round answers to the tenths place.

$t$ (time in minutes)	Linear Equation:	$m$ (distance in miles)
0		
15		
30		
45		
60		

- Graph the data on a coordinate plane.



- Connor ran for 40 minutes before tripping and spraining his ankle. About how many miles did he run before he had to stop? Explain.

## Lesson 12: Linear Equations in Two Variables

### Classwork

#### Opening Exercise

Emily tells you that she scored 32 points in a basketball game. Write down all the possible ways she could have scored 32 with only two- and three-point baskets. Use the table below to organize your work.

Number of Two-Pointers	Number of Three-Pointers

Let  $x$  be the number of two-pointers and  $y$  be the number of three-pointers that Emily scored. Write an equation to represent the situation.

**Exploratory Challenge/Exercises**

1. Find five solutions for the linear equation  $x + y = 3$ , and plot the solutions as points on a coordinate plane.

$x$	Linear Equation: $x + y = 3$	$y$

2. Find five solutions for the linear equation  $2x - y = 10$ , and plot the solutions as points on a coordinate plane.

$x$	Linear Equation: $2x - y = 10$	$y$

3. Find five solutions for the linear equation  $x + 5y = 21$ , and plot the solutions as points on a coordinate plane.

$x$	Linear Equation: $x + 5y = 21$	$y$

4. Consider the linear equation  $\frac{2}{5}x + y = 11$ .
- a. Will you choose to fix values for  $x$  or  $y$ ? Explain.

- b. Are there specific numbers that would make your computational work easier? Explain.

- c. Find five solutions to the linear equation  $\frac{2}{5}x + y = 11$ , and plot the solutions as points on a coordinate plane.

$x$	Linear Equation: $\frac{2}{5}x + y = 11$	$y$

5. At the store, you see that you can buy a bag of candy for \$2 and a drink for \$1. Assume you have a total of \$35 to spend. You are feeling generous and want to buy some snacks for you and your friends.
- a. Write an equation in standard form to represent the number of bags of candy,  $x$ , and the number of drinks,  $y$ , that you can buy with \$35.

- b. Find five solutions to the linear equation from part (a), and plot the solutions as points on a coordinate plane.

$x$	Linear Equation:	$y$

**Lesson Summary**

A linear equation in two-variables  $x$  and  $y$  is in standard form if it is of the form  $ax + by = c$  for numbers  $a$ ,  $b$ , and  $c$ , where  $a$  and  $b$  are both not zero. The numbers  $a$ ,  $b$ , and  $c$  are called constants.

A solution to a linear equation in two variables is the ordered pair  $(x, y)$  that makes the given equation true. Solutions can be found by fixing a number for  $x$  and solving for  $y$  or fixing a number for  $y$  and solving for  $x$ .

**Problem Set**

- Consider the linear equation  $x - \frac{3}{2}y = -2$ .
  - Will you choose to fix values for  $x$  or  $y$ ? Explain.
  - Are there specific numbers that would make your computational work easier? Explain.
  - Find five solutions to the linear equation  $x - \frac{3}{2}y = -2$ , and plot the solutions as points on a coordinate plane.

$x$	Linear Equation: $x - \frac{3}{2}y = -2$	$y$

- Find five solutions for the linear equation  $\frac{1}{3}x + y = 12$ , and plot the solutions as points on a coordinate plane.
- Find five solutions for the linear equation  $-x + \frac{3}{4}y = -6$ , and plot the solutions as points on a coordinate plane.
- Find five solutions for the linear equation  $2x + y = 5$ , and plot the solutions as points on a coordinate plane.
- Find five solutions for the linear equation  $3x - 5y = 15$ , and plot the solutions as points on a coordinate plane.

## Lesson 13: The Graph of a Linear Equation in Two Variables

### Classwork

#### Exercises

1. Find at least ten solutions to the linear equation  $3x + y = -8$ , and plot the points on a coordinate plane.

$x$	Linear Equation: $3x + y = -8$	$y$

What shape is the graph of the linear equation taking?

2. Find at least ten solutions to the linear equation  $x - 5y = 11$ , and plot the points on a coordinate plane.

$x$	Linear Equation: $x - 5y = 11$	$y$

What shape is the graph of the linear equation taking?

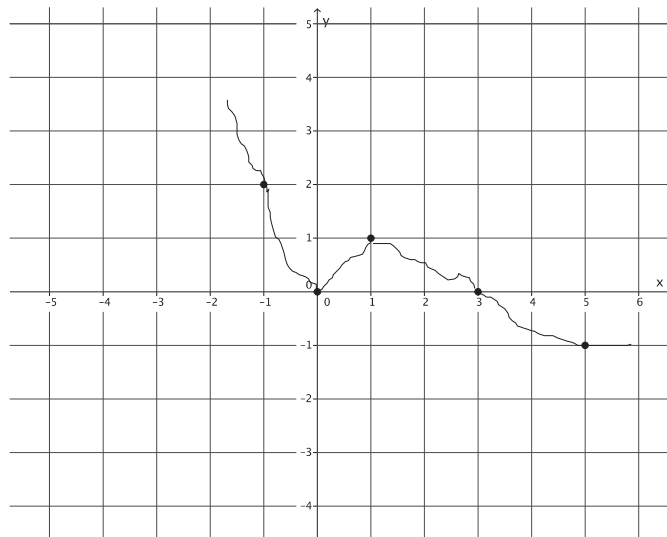
3. Compare the solutions you found in Exercise 1 with a partner. Add the partner's solutions to your graph.

Is the prediction you made about the shape of the graph still true? Explain.

4. Compare the solutions you found in Exercise 2 with a partner. Add the partner's solutions to your graph.

Is the prediction you made about the shape of the graph still true? Explain.

5. Joey predicts that the graph of  $-x + 2y = 3$  will look like the graph shown below. Do you agree? Explain why or why not.



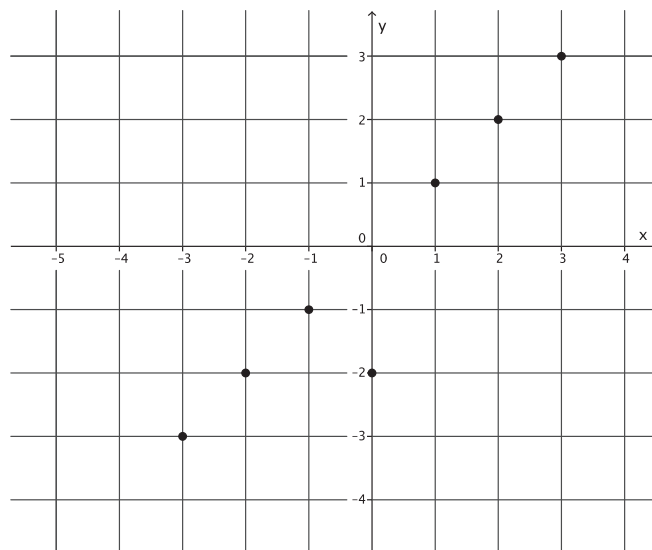
6. We have looked at some equations that appear to be lines. Can you write an equation that has solutions that do not form a line? Try to come up with one, and prove your assertion on the coordinate plane.

**Lesson Summary**

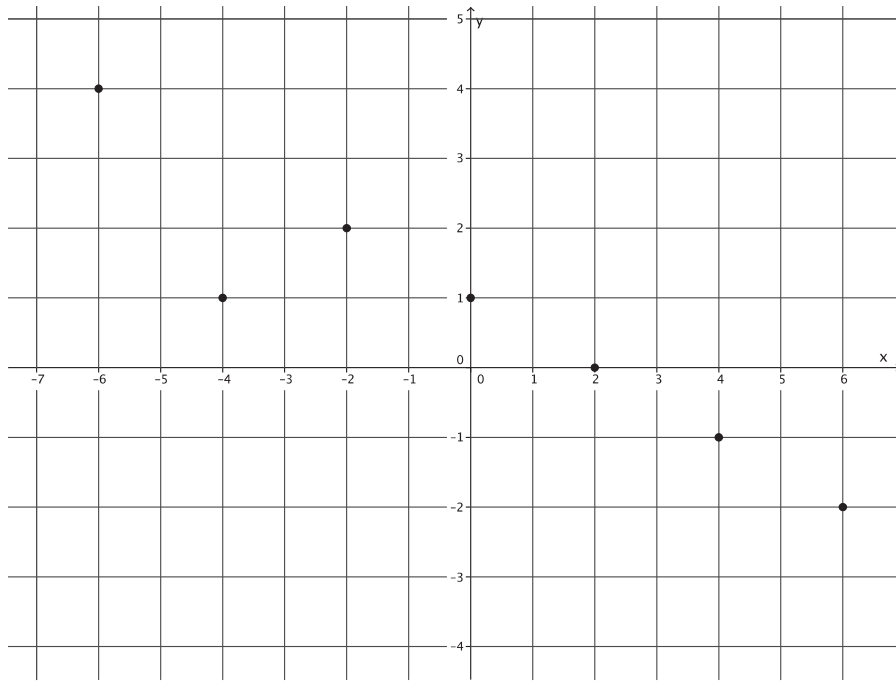
One way to determine if a given point is on the graph of a linear equation is by checking to see if it is a solution to the equation. Note that all graphs of linear equations appear to be lines.

**Problem Set**

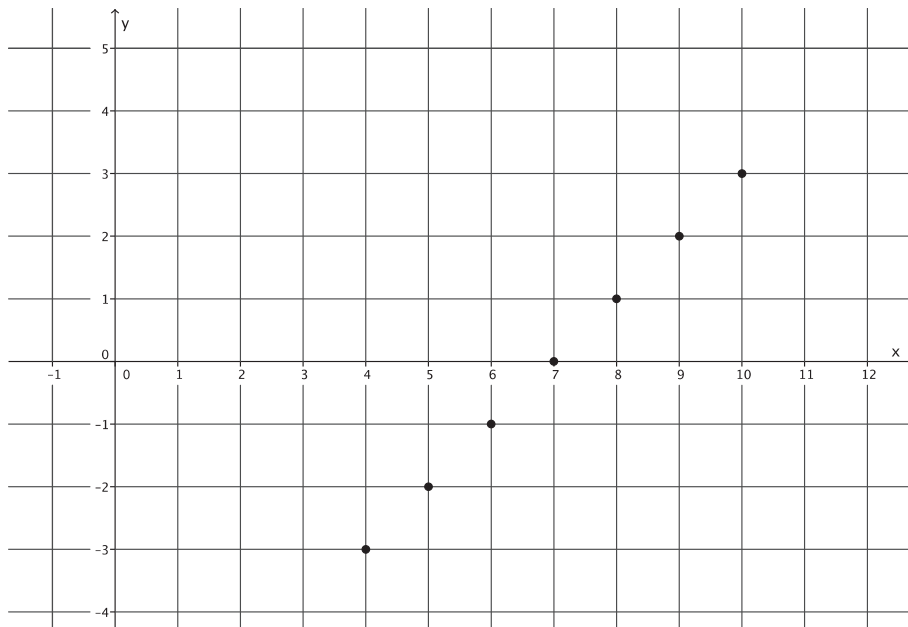
1. Find at least ten solutions to the linear equation  $\frac{1}{2}x + y = 5$ , and plot the points on a coordinate plane.  
What shape is the graph of the linear equation taking?
2. Can the following points be on the graph of the equation  $x - y = 0$ ? Explain.



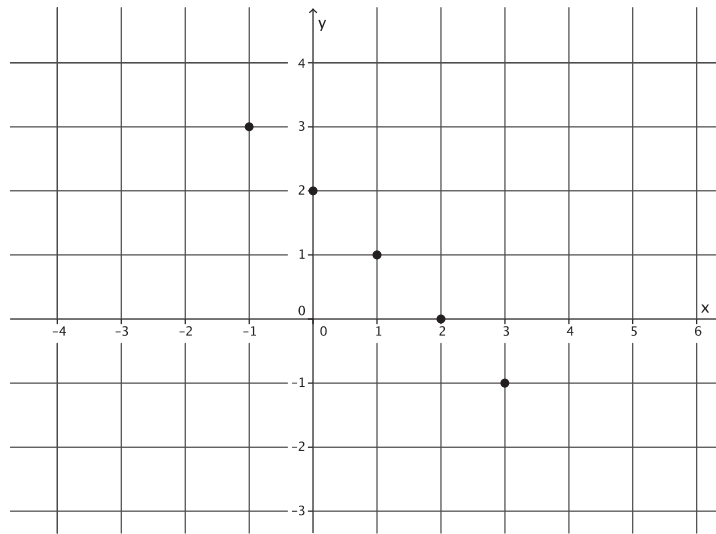
3. Can the following points be on the graph of the equation  $x + 2y = 2$ ? Explain.



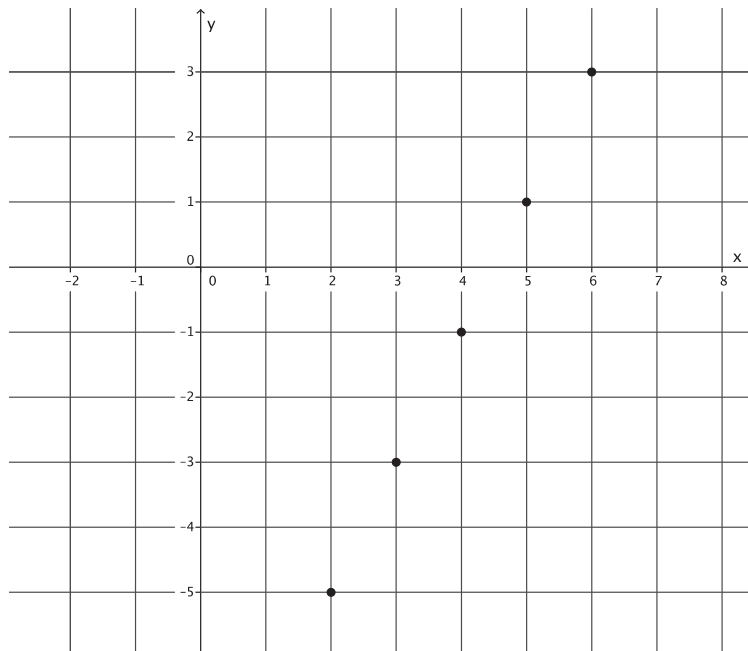
4. Can the following points be on the graph of the equation  $x - y = 7$ ? Explain.



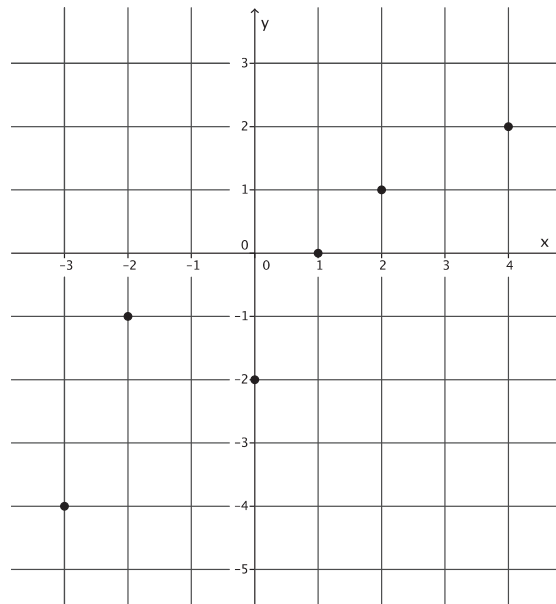
5. Can the following points be on the graph of the equation  $x + y = 2$ ? Explain.



6. Can the following points be on the graph of the equation  $2x - y = 9$ ? Explain.



7. Can the following points be on the graph of the equation  $x - y = 1$ ? Explain.



## Lesson 14: The Graph of a Linear Equation—Horizontal and Vertical Lines

### Classwork

#### Exercises

1. Find at least four solutions to graph the linear equation  $1x + 2y = 5$ .
2. Find at least four solutions to graph the linear equation  $1x + 0y = 5$ .
3. What was different about the equations in Exercises 1 and 2? What effect did this change have on the graph?
4. Graph the linear equation  $x = -2$ .
5. Graph the linear equation  $x = 3$ .
6. What will the graph of  $x = 0$  look like?
7. Find at least four solutions to graph the linear equation  $2x + 1y = 2$ .
8. Find at least four solutions to graph the linear equation  $0x + 1y = 2$ .
9. What was different about the equations in Exercises 7 and 8? What effect did this change have on the graph?
10. Graph the linear equation  $y = -2$ .
11. Graph the linear equation  $y = 3$ .
12. What will the graph of  $y = 0$  look like?

**Lesson Summary**

In a coordinate plane with perpendicular  $x$ - and  $y$ -axes, a *vertical line* is either the  $y$ -axis or any other line parallel to the  $y$ -axis. The graph of the linear equation in two variables  $ax + by = c$ , where  $a = 1$  and  $b = 0$ , is the graph of the equation  $x = c$ . The graph of  $x = c$  is the vertical line that passes through the point  $(c, 0)$ .

In a coordinate plane with perpendicular  $x$ - and  $y$ -axes, a *horizontal line* is either the  $x$ -axis or any other line parallel to the  $x$ -axis. The graph of the linear equation in two variables  $ax + by = c$ , where  $a = 0$  and  $b = 1$ , is the graph of the equation  $y = c$ . The graph of  $y = c$  is the horizontal line that passes through the point  $(0, c)$ .

**Problem Set**

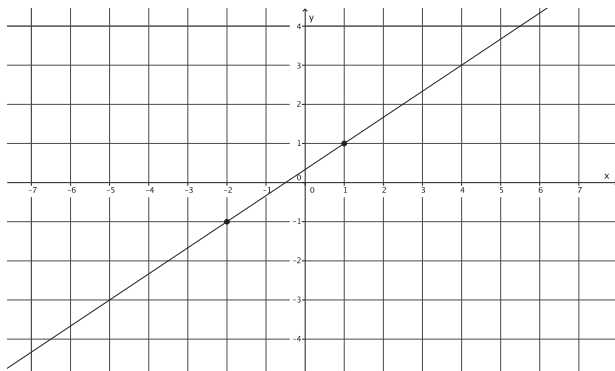
1. Graph the two-variable linear equation  $ax + by = c$ , where  $a = 0$ ,  $b = 1$ , and  $c = -4$ .
2. Graph the two-variable linear equation  $ax + by = c$ , where  $a = 1$ ,  $b = 0$ , and  $c = 9$ .
3. Graph the linear equation  $y = 7$ .
4. Graph the linear equation  $x = 1$ .
5. Explain why the graph of a linear equation in the form of  $y = c$  is the horizontal line, parallel to the  $x$ -axis passing through the point  $(0, c)$ .
6. Explain why there is only one line with the equation  $y = c$  that passes through the point  $(0, c)$ .

## Lesson 15: The Slope of a Non-Vertical Line

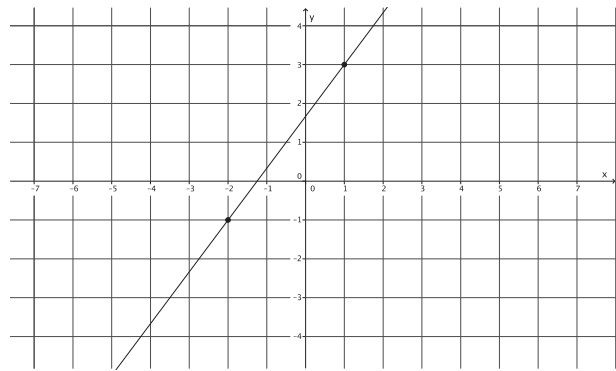
### Classwork

#### Opening Exercise

Graph A



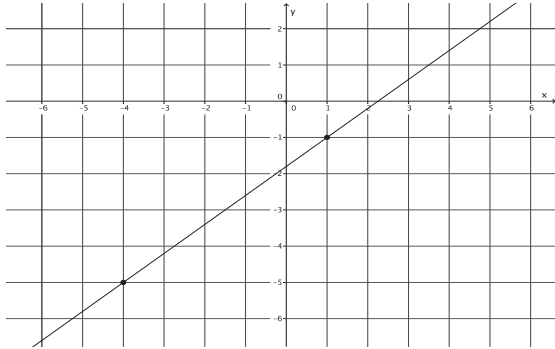
Graph B



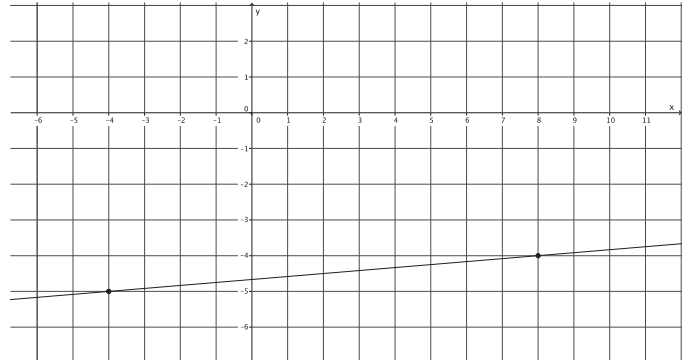
- Which graph is steeper?
- Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.
- Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)?

Pair 1:

Graph A



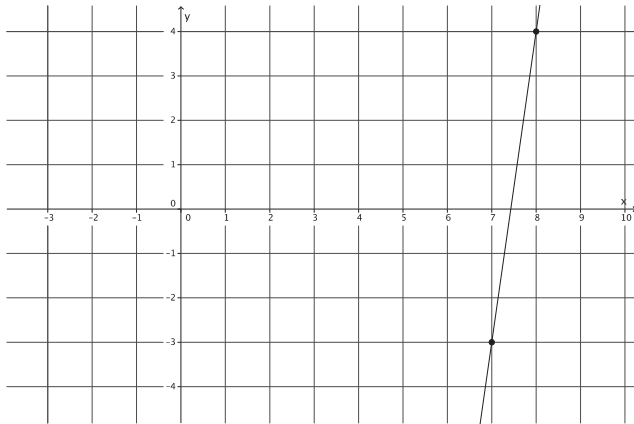
Graph B



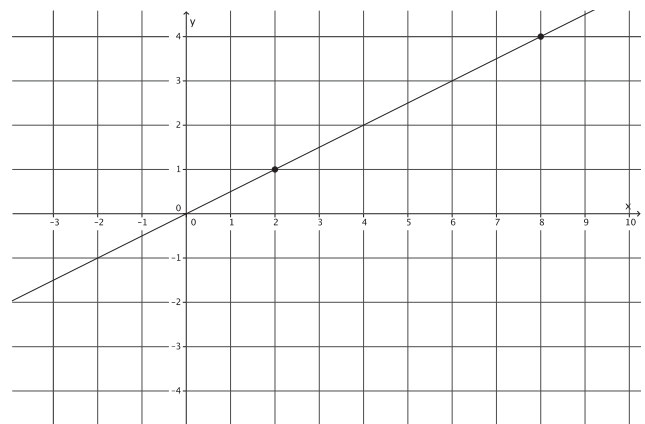
- Which graph is steeper?
- Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.
- Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)?

Pair 2:

Graph A



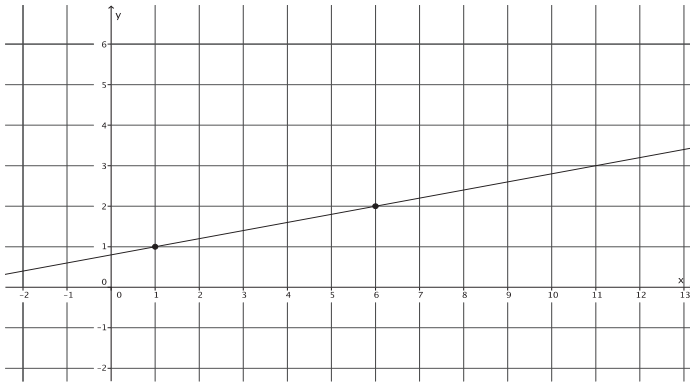
Graph B



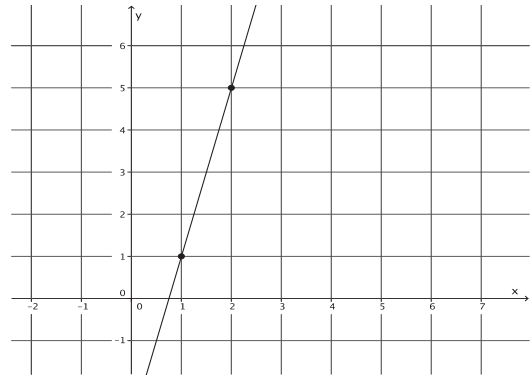
- Which graph is steeper?
- Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.
- Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)?

Pair 3:

Graph A



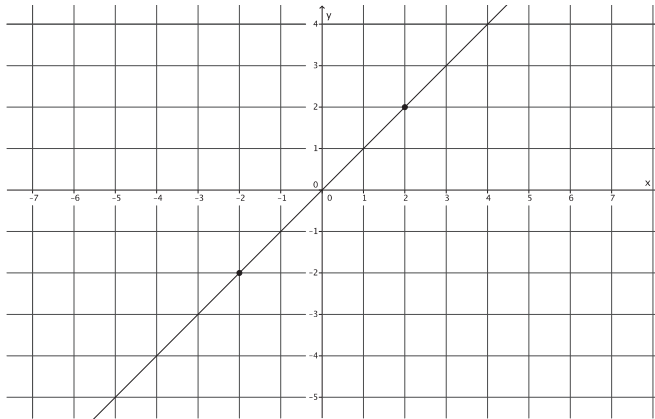
Graph B



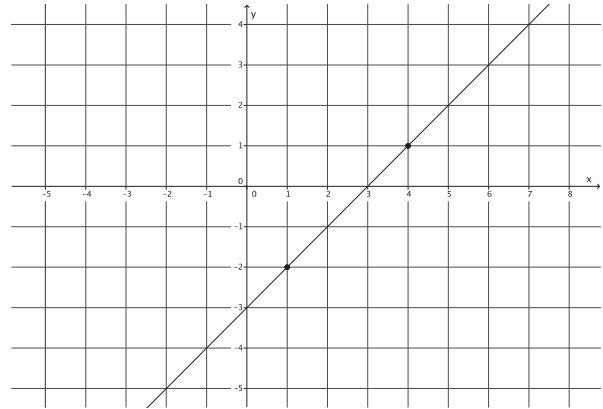
- Which graph is steeper?
- Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.
- Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)?

Pair 4:

Graph A



Graph B

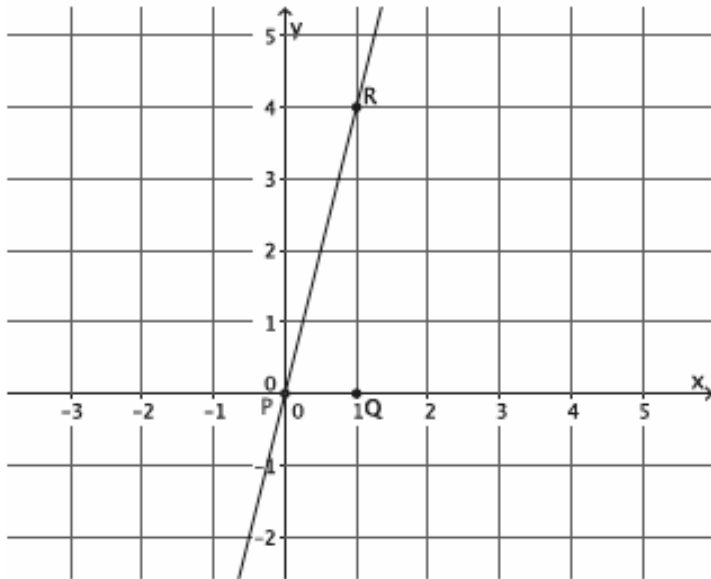


- Which graph is steeper?
- Write directions that explain how to move from one point on the graph to the other for both Graph A and Graph B.
- Write the directions from part (b) as ratios, and then compare the ratios. How does this relate to which graph was steeper in part (a)?

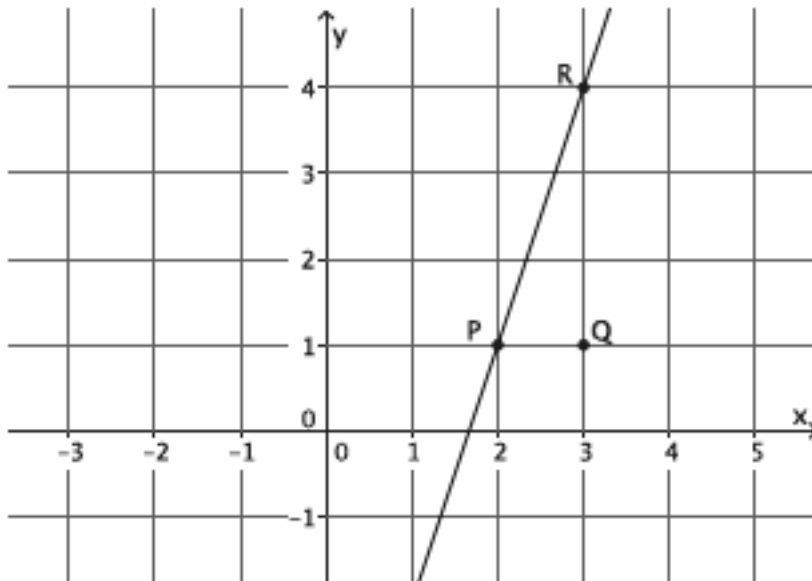
**Exercises**

Use your transparency to find the slope of each line if needed.

1. What is the slope of this non-vertical line?

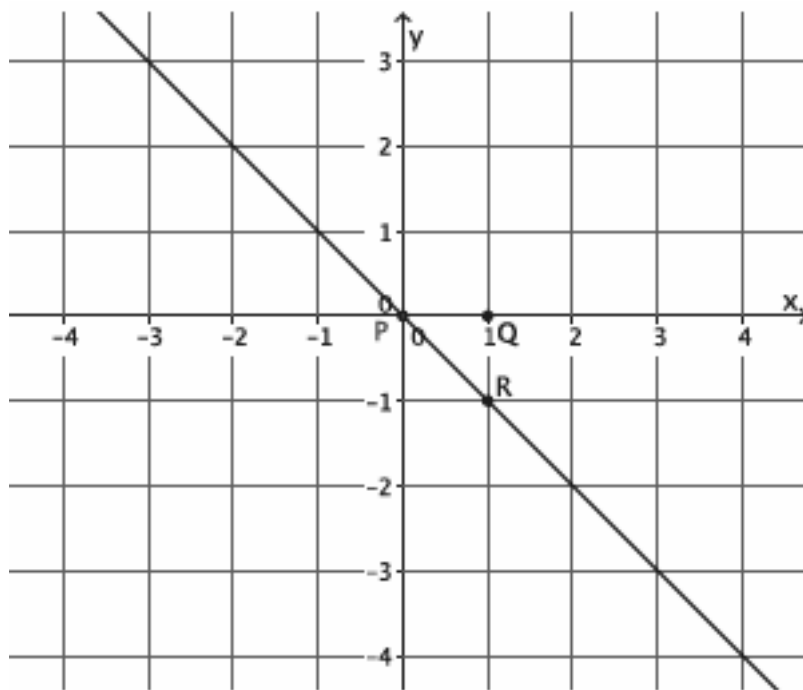


2. What is the slope of this non-vertical line?

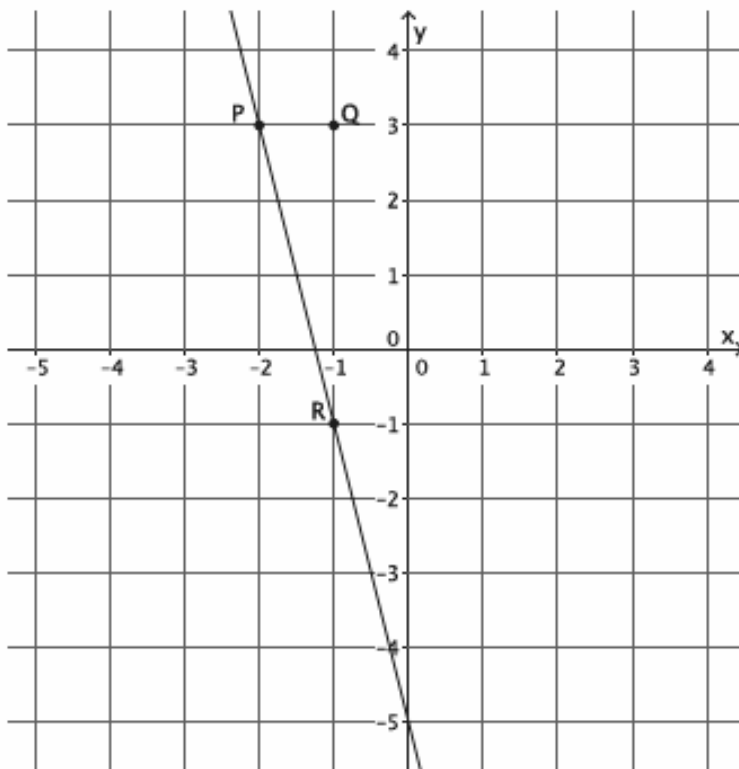


3. Which of the lines in Exercises 1 and 2 is steeper? Compare the slopes of each of the lines. Is there a relationship between steepness and slope?

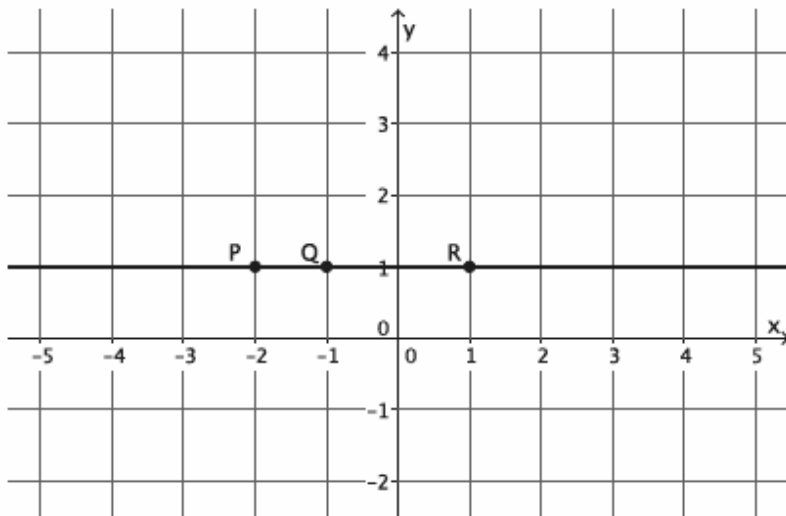
4. What is the slope of this non-vertical line?



5. What is the slope of this non-vertical line?



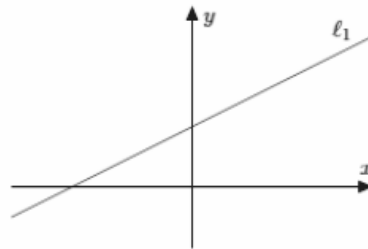
6. What is the slope of this non-vertical line?



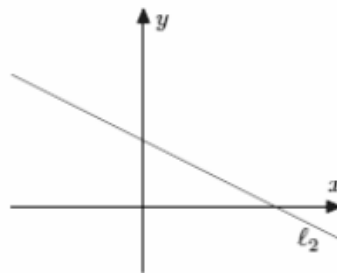
**Lesson Summary**

Slope is a number that can be used to describe the steepness of a line in a coordinate plane. The slope of a line is often represented by the symbol  $m$ .

Lines in a coordinate plane that are *left-to-right inclining* have a positive slope, as shown below.

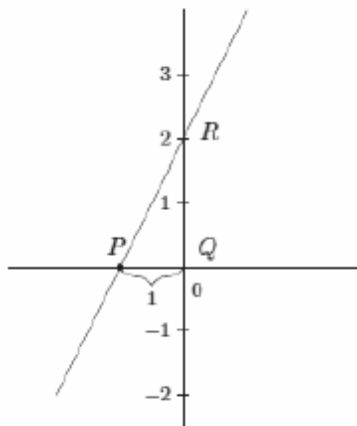


Lines in a coordinate plane that are *left-to-right declining* have a negative slope, as shown below.



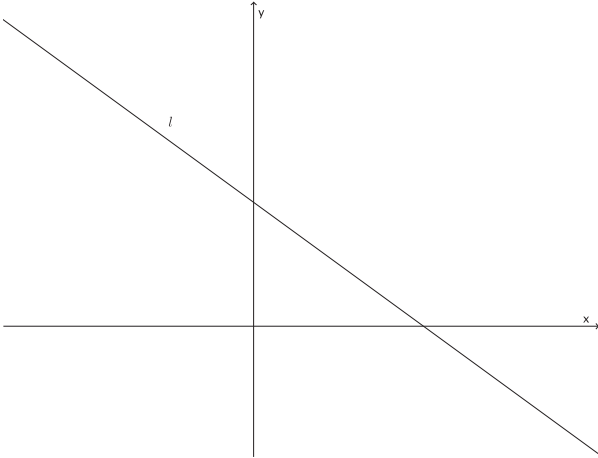
Determine the slope of a line when the horizontal distance between points is fixed at 1 by translating point  $Q$  to the origin of the graph and then identifying the  $y$ -coordinate of point  $R$ ; by definition, that number is the slope of the line.

The slope of the line shown below is 2, so  $m = 2$ , because point  $R$  is at 2 on the  $y$ -axis.

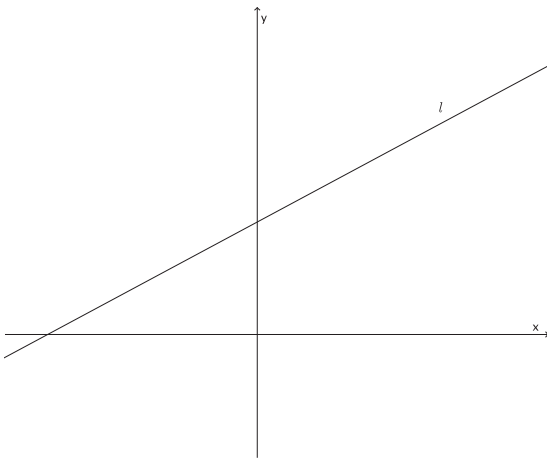


**Problem Set**

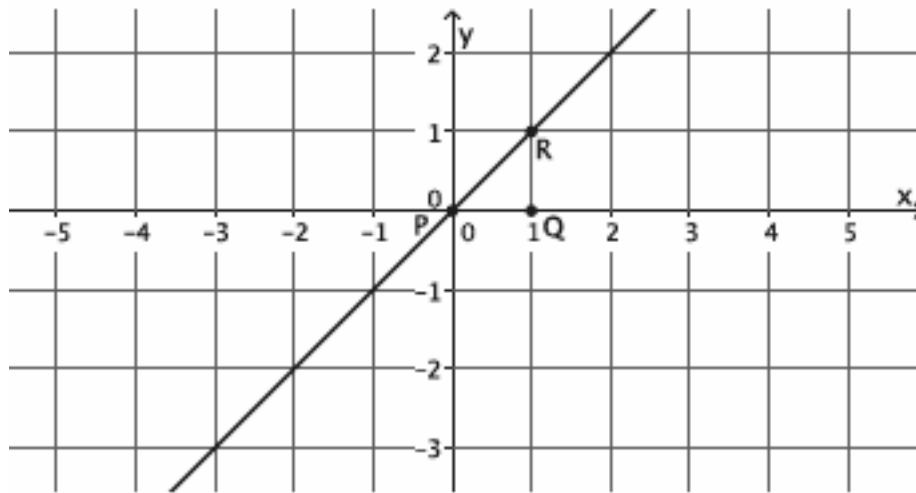
1. Does the graph of the line shown below have a positive or negative slope? Explain.



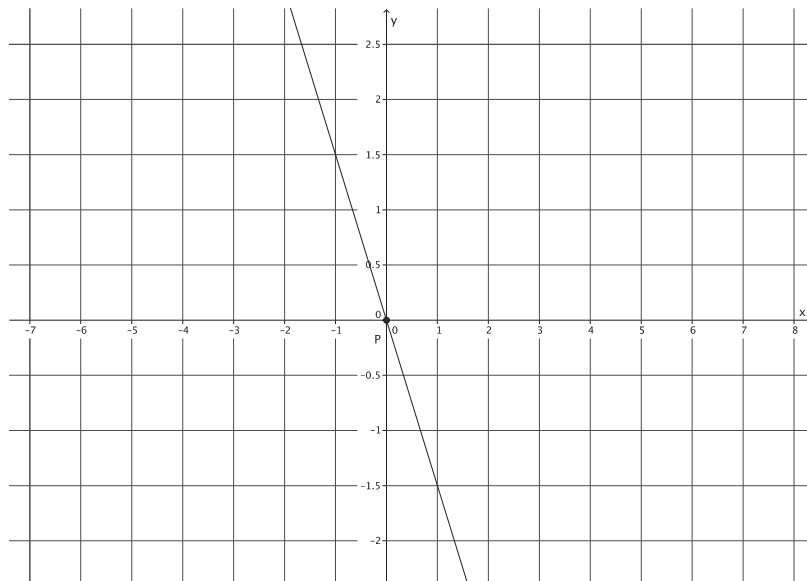
2. Does the graph of the line shown below have a positive or negative slope? Explain.



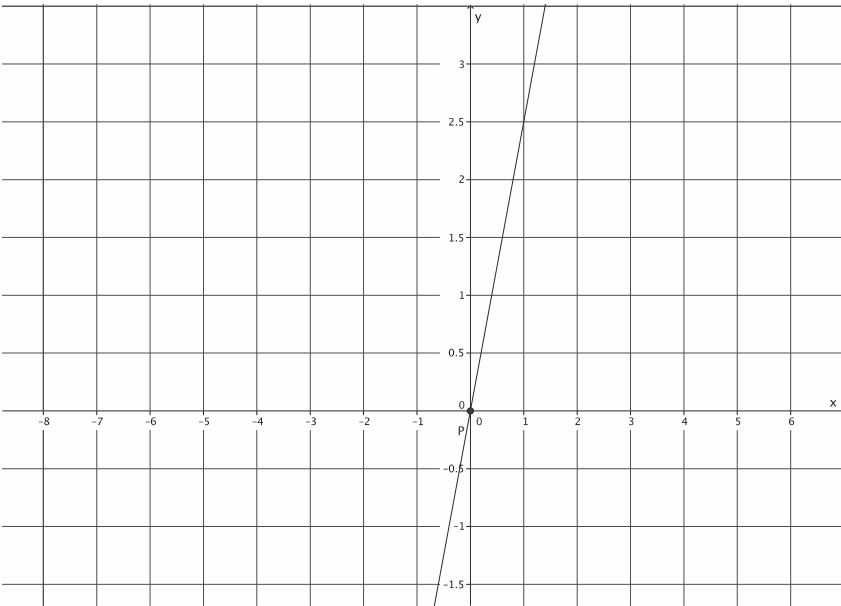
3. What is the slope of this non-vertical line? Use your transparency if needed.



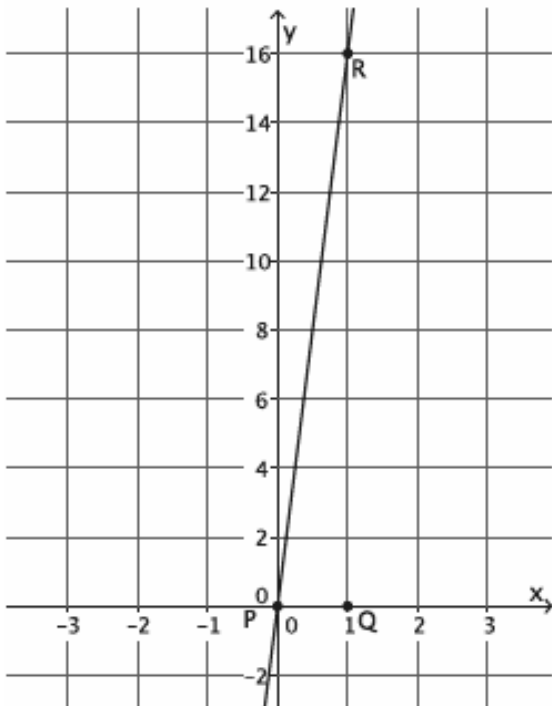
4. What is the slope of this non-vertical line? Use your transparency if needed.



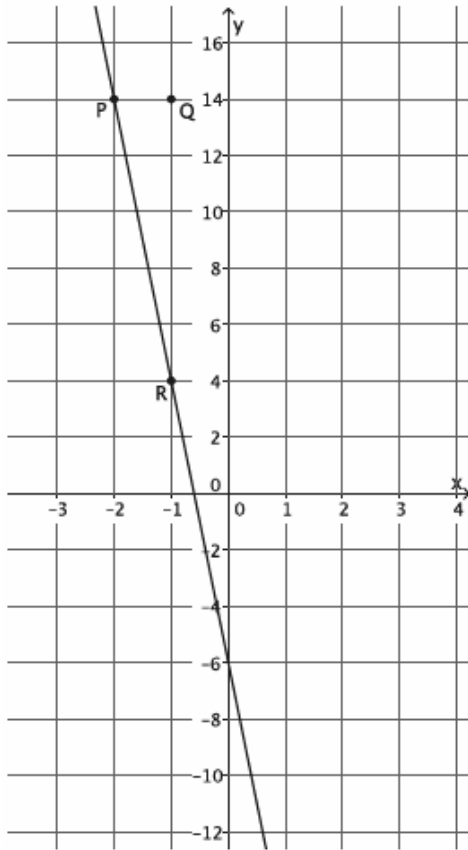
5. What is the slope of this non-vertical line? Use your transparency if needed.



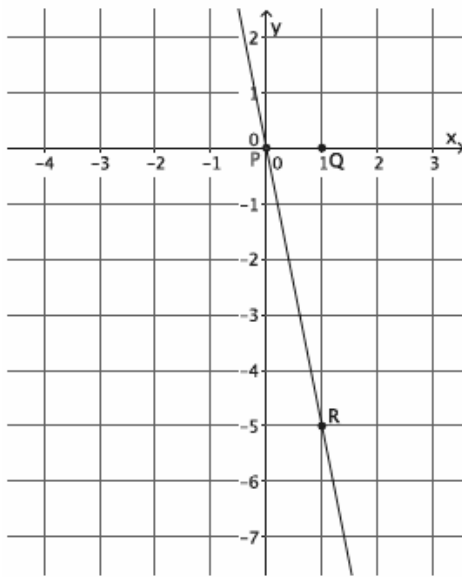
6. What is the slope of this non-vertical line? Use your transparency if needed.



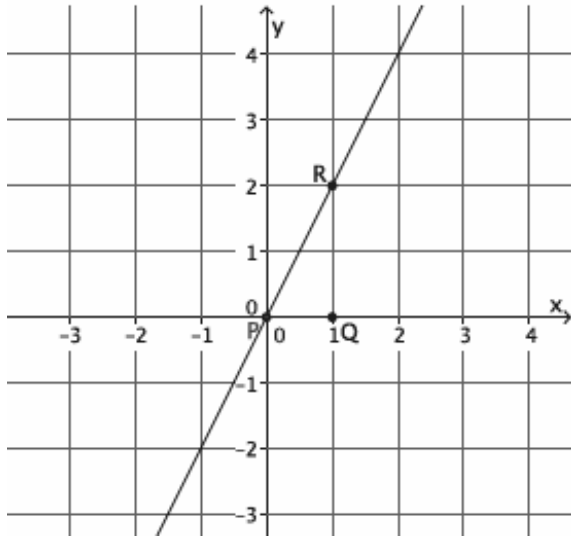
7. What is the slope of this non-vertical line? Use your transparency if needed.



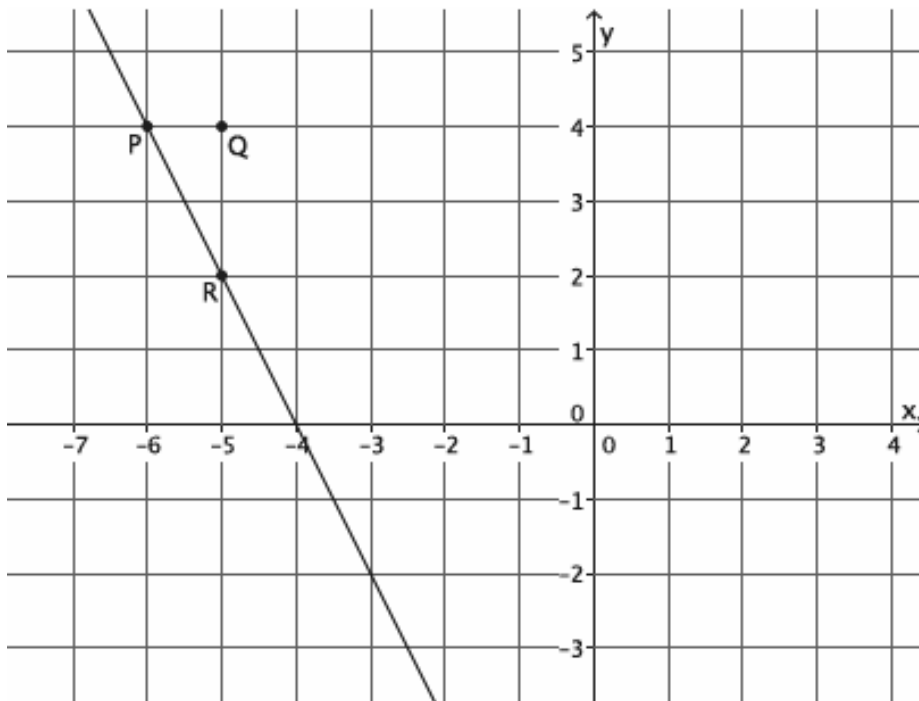
8. What is the slope of this non-vertical line? Use your transparency if needed.



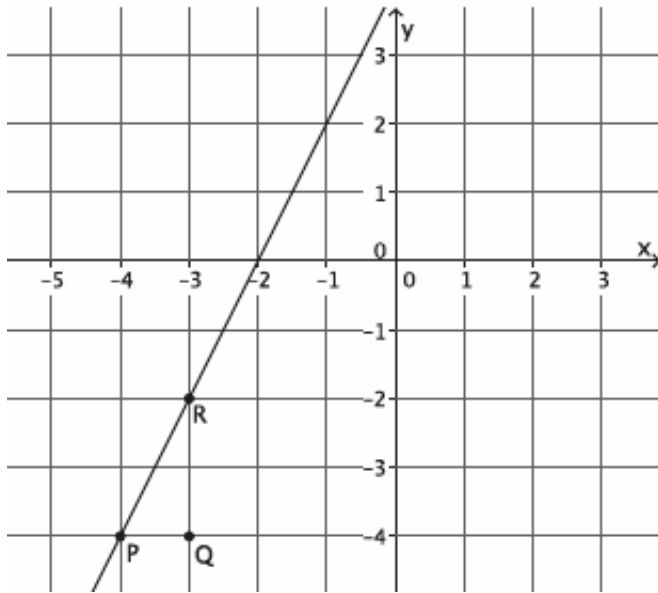
9. What is the slope of this non-vertical line? Use your transparency if needed.



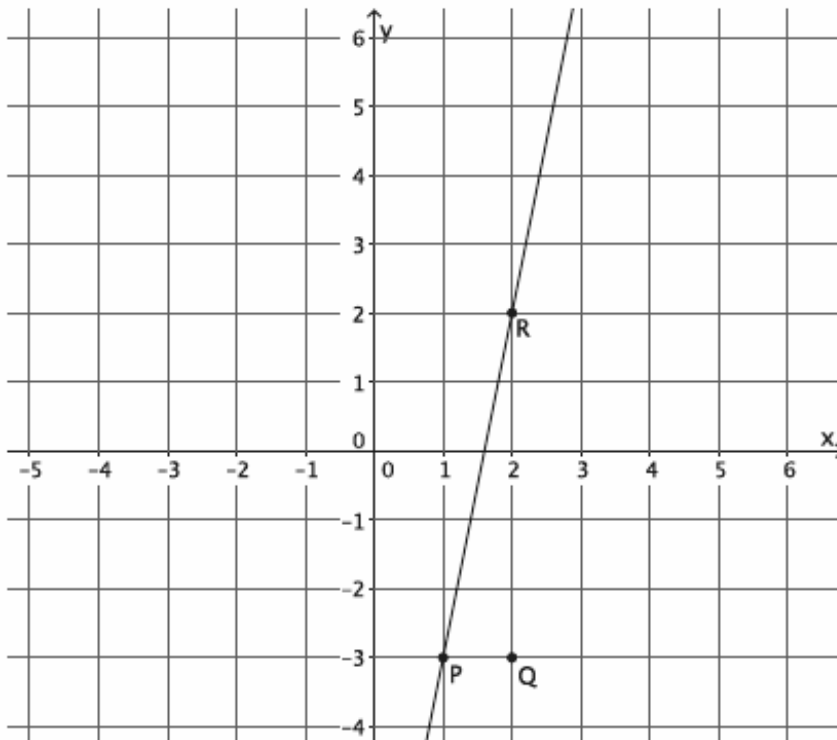
10. What is the slope of this non-vertical line? Use your transparency if needed.



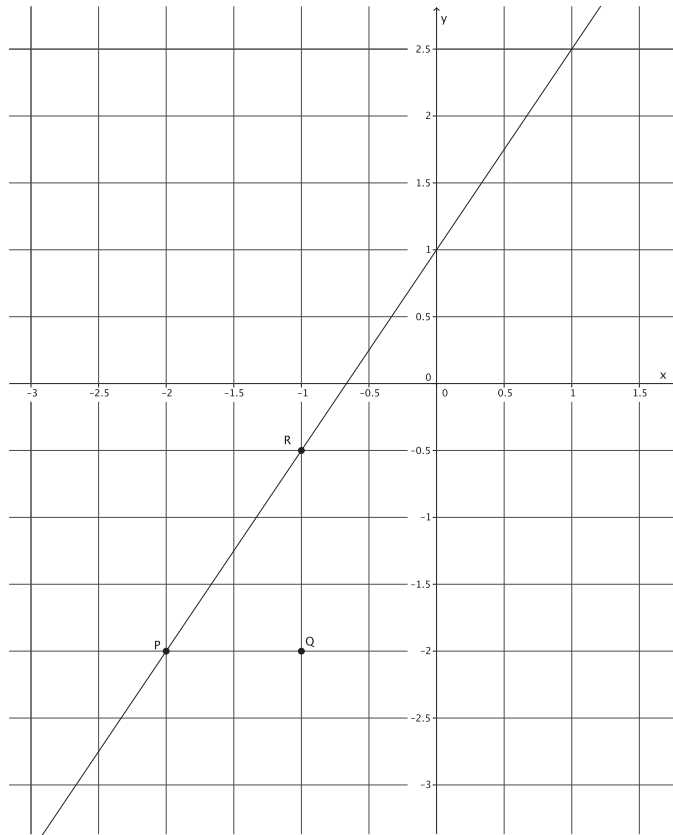
11. What is the slope of this non-vertical line? Use your transparency if needed.



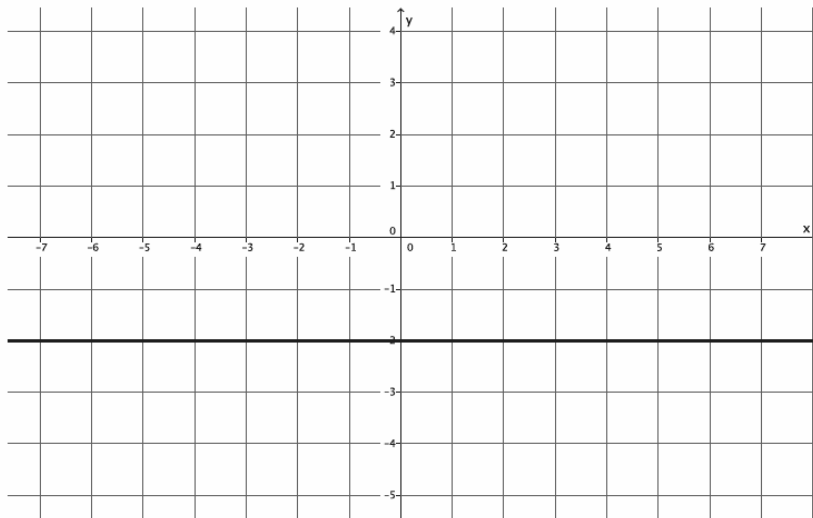
12. What is the slope of this non-vertical line? Use your transparency if needed.



13. What is the slope of this non-vertical line? Use your transparency if needed.



14. What is the slope of this non-vertical line? Use your transparency if needed.

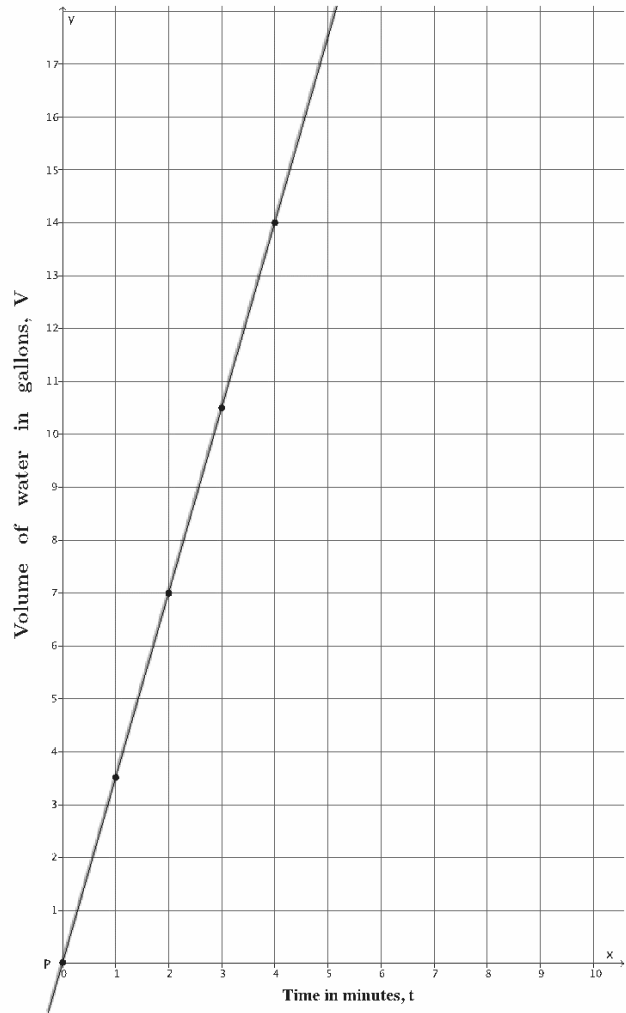


In Lesson 11, you did the work below involving constant rate problems. Use the table and the graphs provided to answer the questions that follow.

15. Suppose the volume of water that comes out in three minutes is 10.5 gallons.

$t$ (time in minutes)	Linear Equation: $V = \frac{10.5}{3}t$	$V$ (in gallons)
0	$V = \frac{10.5}{3}(0)$	0
1	$V = \frac{10.5}{3}(1)$	$\frac{10.5}{3} = 3.5$
2	$V = \frac{10.5}{3}(2)$	$\frac{21}{3} = 7$
3	$V = \frac{10.5}{3}(3)$	$\frac{31.5}{3} = 10.5$
4	$V = \frac{10.5}{3}(4)$	$\frac{42}{3} = 14$

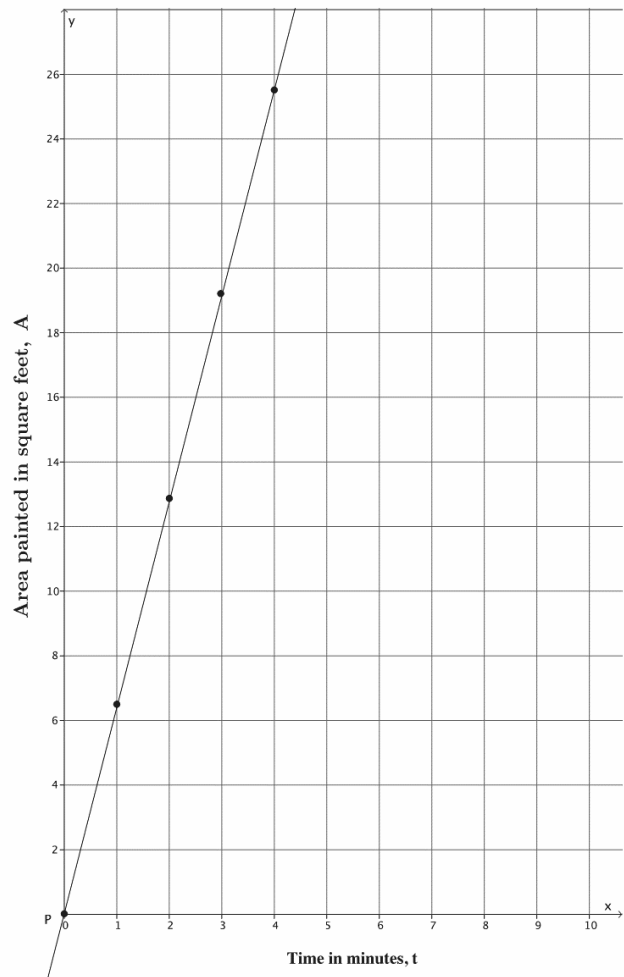
- How many gallons of water flow out of the faucet per minute? In other words, what is the unit rate of water flow?
- Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?



16. Emily paints at a constant rate. She can paint 32 square feet in five minutes.

$t$ (time in minutes)	Linear Equation: $A = \frac{32}{5}t$	$A$ (area painted in square feet)
0	$A = \frac{32}{5}(0)$	0
1	$A = \frac{32}{5}(1)$	$\frac{32}{5} = 6.4$
2	$A = \frac{32}{5}(2)$	$\frac{64}{5} = 12.8$
3	$A = \frac{32}{5}(3)$	$\frac{96}{5} = 19.2$
4	$A = \frac{32}{5}(4)$	$\frac{128}{5} = 25.6$

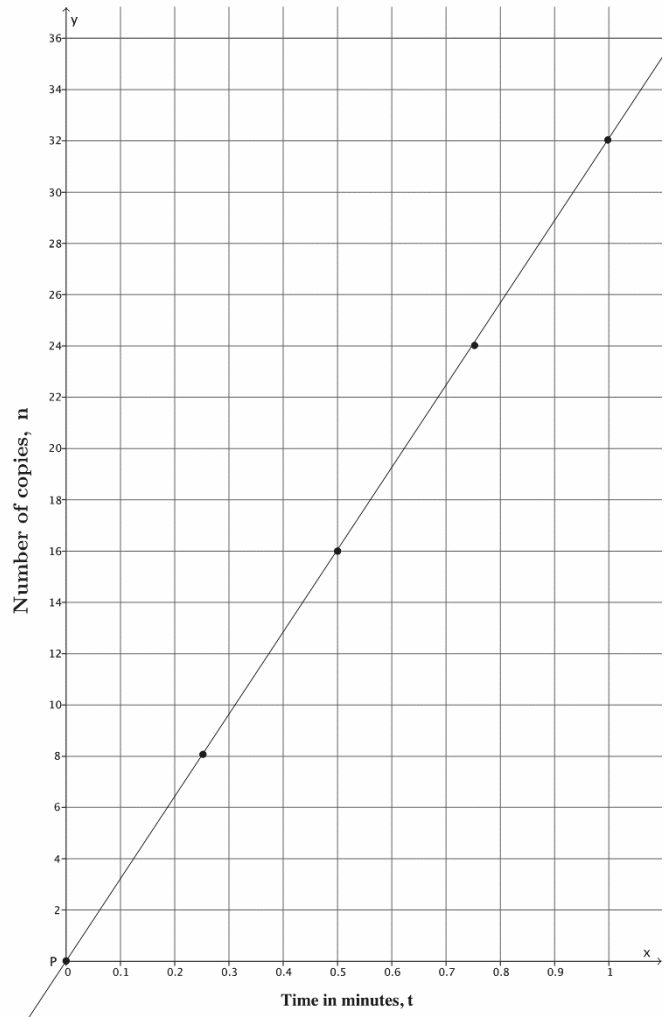
- How many square feet can Emily paint in one minute? In other words, what is her unit rate of painting?
- Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?



17. A copy machine makes copies at a constant rate. The machine can make 80 copies in  $2\frac{1}{2}$  minutes.

$t$ (time in minutes)	Linear Equation: $n = 32t$	$n$ (number of copies)
0	$n = 32(0)$	0
0.25	$n = 32(0.25)$	8
0.5	$n = 32(0.5)$	16
0.75	$n = 32(0.75)$	24
1	$n = 32(1)$	32

- How many copies can the machine make each minute? In other words, what is the unit rate of the copy machine?
- Assume that the graph of the situation is a line, as shown in the graph. What is the slope of the line?

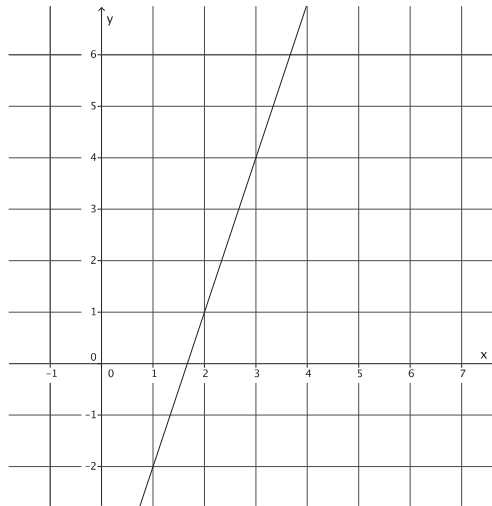


## Lesson 16: The Computation of the Slope of a Non-Vertical Line

### Classwork

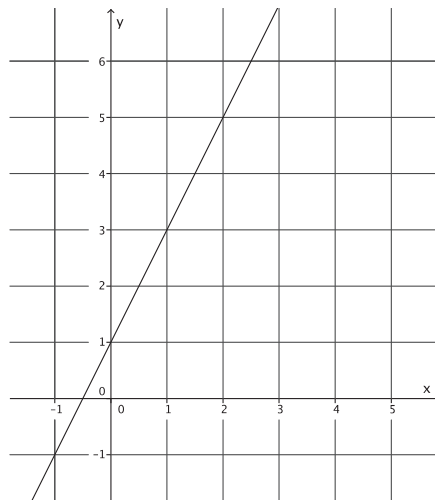
#### Example 1

Using what you learned in the last lesson, determine the slope of the line with the following graph.



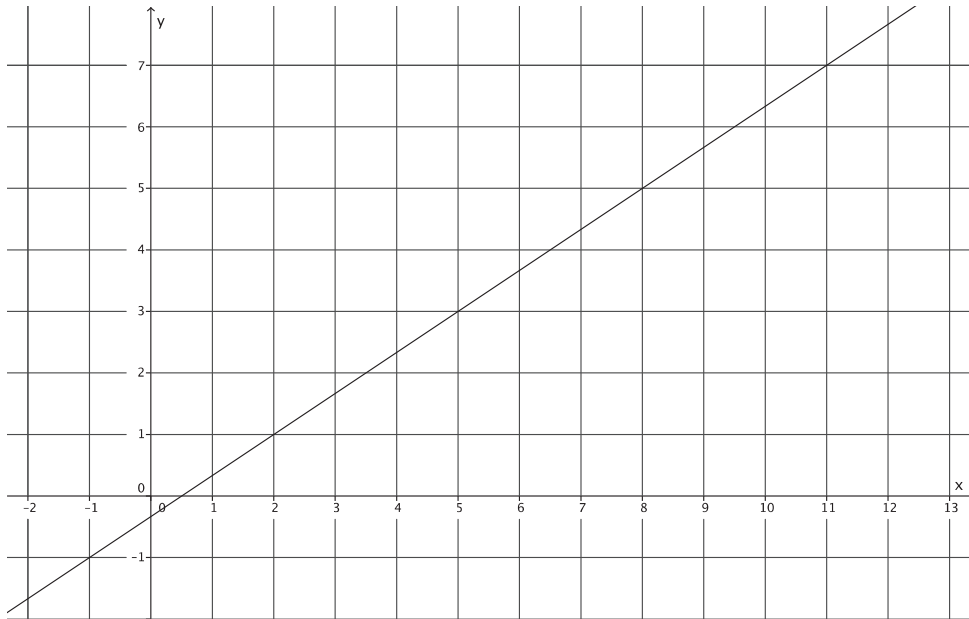
#### Example 2

Using what you learned in the last lesson, determine the slope of the line with the following graph.



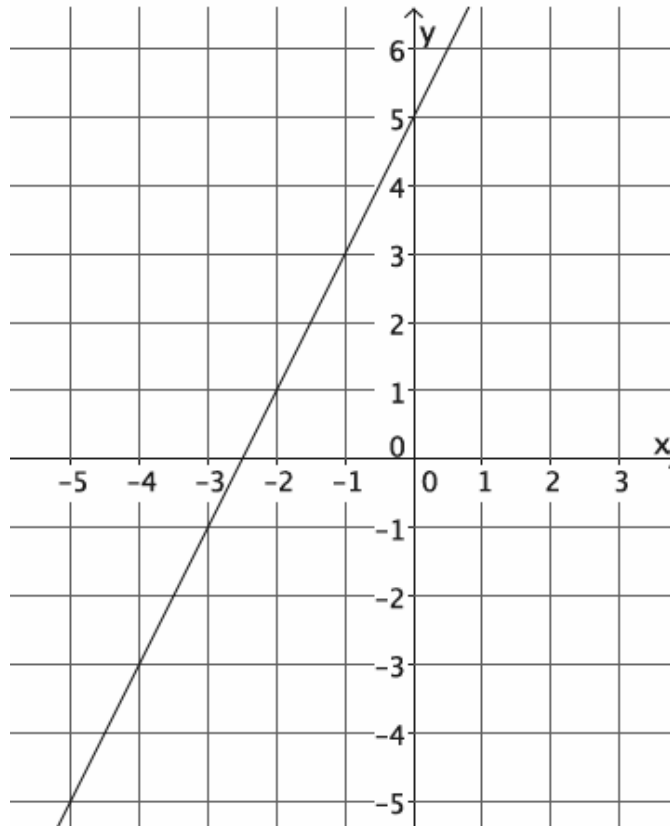
**Example 3**

What is different about this line compared to the last two examples?



**Exercise**

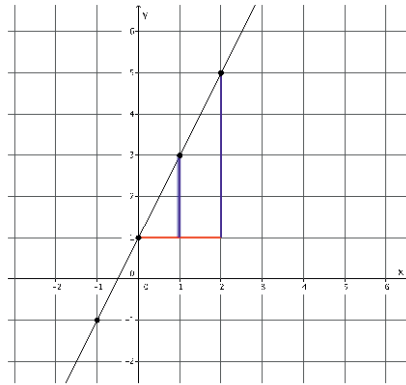
Let's investigate concretely to see if the claim that we can find slope between any two points is true.



- Select any two points on the line to label as  $P$  and  $R$ .
- Identify the coordinates of points  $P$  and  $R$ .
- Find the slope of the line using as many different points as you can. Identify your points, and show your work below.

### Lesson Summary

The slope of a line can be calculated using *any* two points on the same line because the slope triangles formed are similar, and corresponding sides will be equal in ratio.



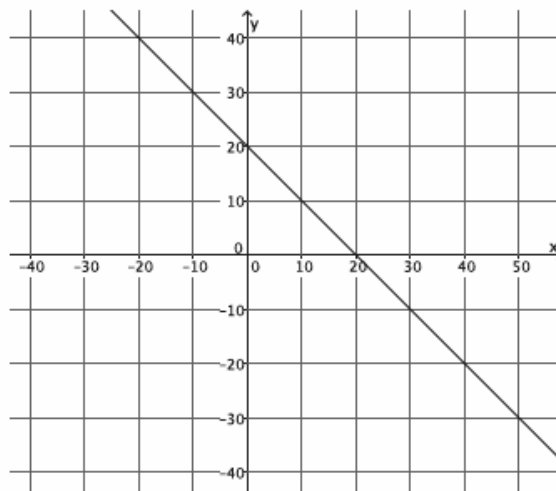
The *slope* of a non-vertical line in a coordinate plane that passes through two different points is the number given by the difference in  $y$ -coordinates of those points divided by the difference in the corresponding  $x$ -coordinates. For two points  $P(p_1, p_2)$  and  $R(r_1, r_2)$  on the line where  $p_1 \neq r_1$ , the slope of the line  $m$  can be computed by the formula

$$m = \frac{p_2 - r_2}{p_1 - r_1}.$$

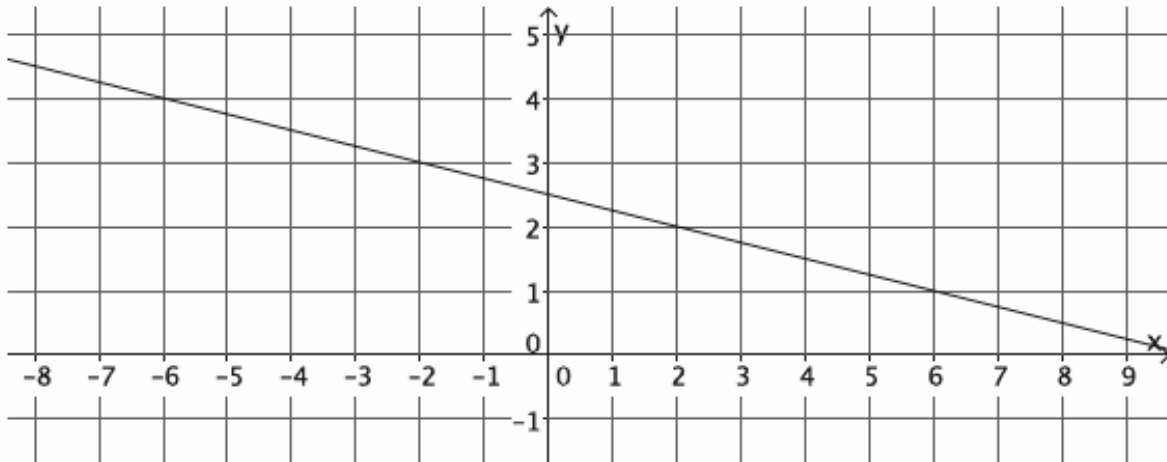
The slope of a vertical line is not defined.

### Problem Set

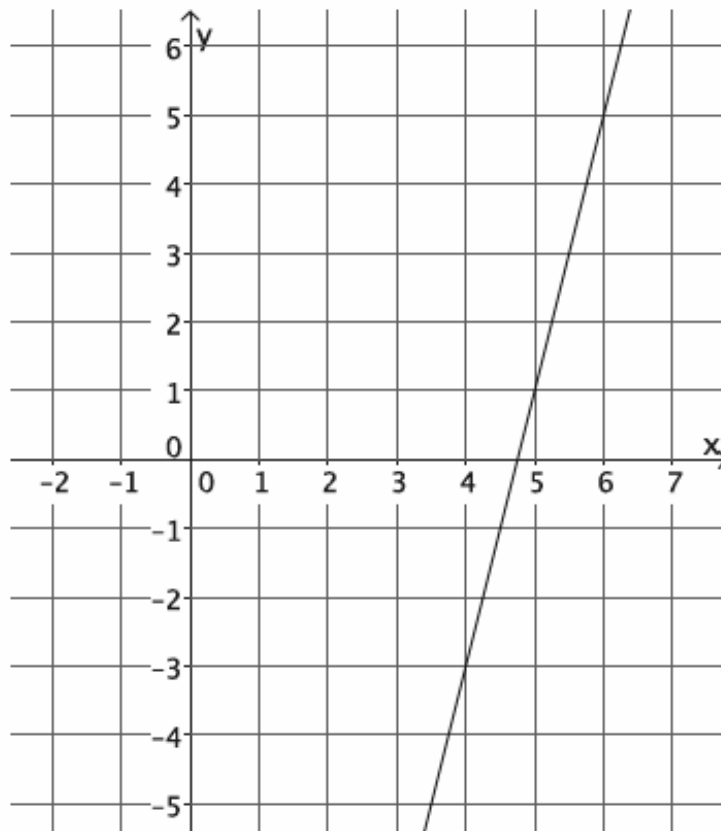
- Calculate the slope of the line using two different pairs of points.



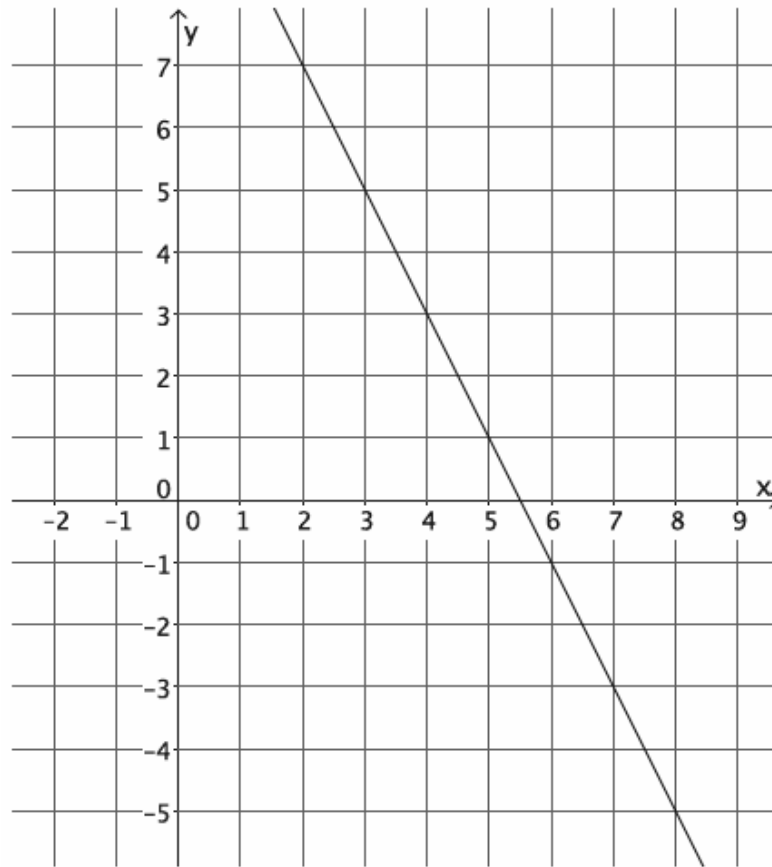
2. Calculate the slope of the line using two different pairs of points.



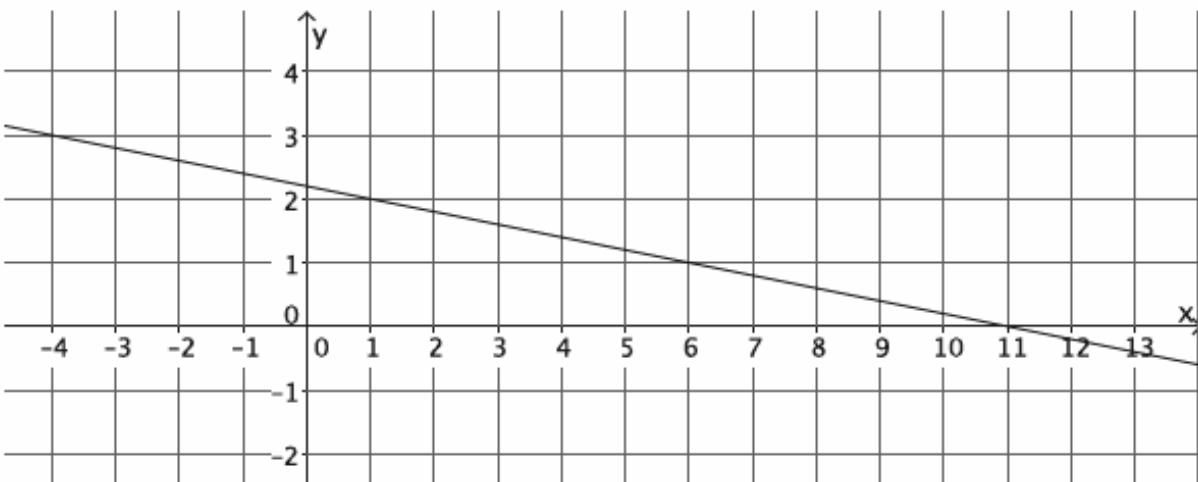
3. Calculate the slope of the line using two different pairs of points.



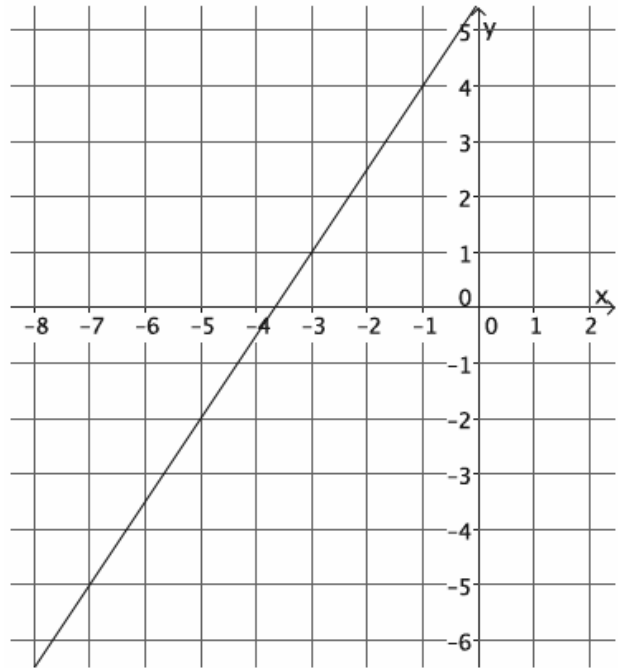
4. Calculate the slope of the line using two different pairs of points.



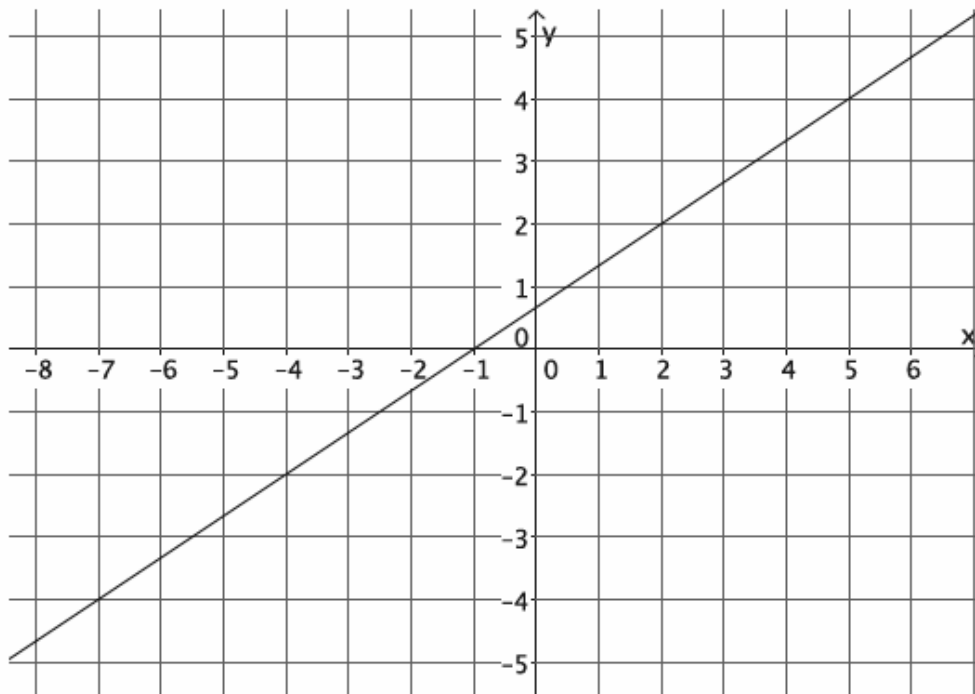
5. Calculate the slope of the line using two different pairs of points.



6. Calculate the slope of the line using two different pairs of points.
- Select any two points on the line to compute the slope.
  - Select two different points on the line to calculate the slope.
  - What do you notice about your answers in parts (a) and (b)? Explain.



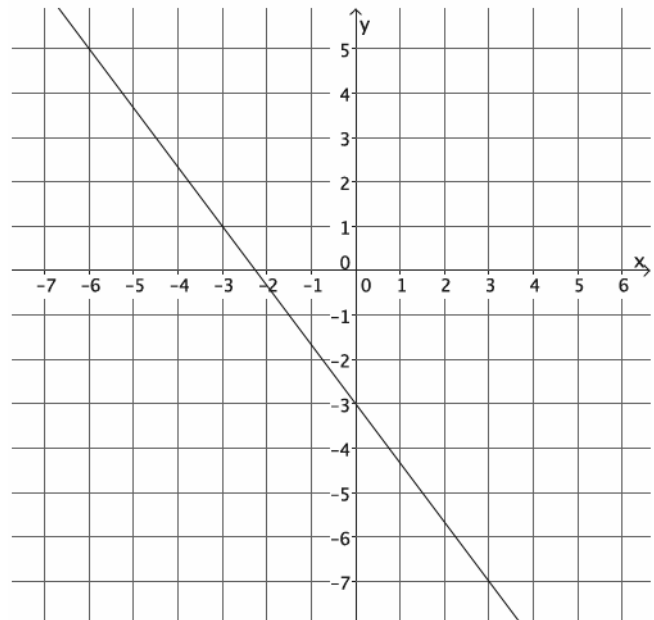
7. Calculate the slope of the line in the graph below.



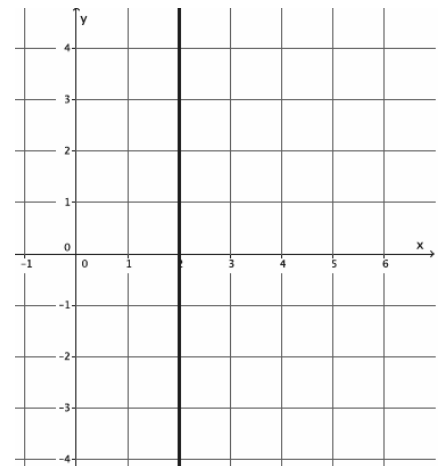
8. Your teacher tells you that a line goes through the points  $(-6, \frac{1}{2})$  and  $(-4, 3)$ .
- Calculate the slope of this line.
  - Do you think the slope will be the same if the order of the points is reversed? Verify by calculating the slope, and explain your result.

9. Use the graph to complete parts (a)–(c).

- Select any two points on the line to calculate the slope.
- Compute the slope again, this time reversing the order of the coordinates.
- What do you notice about the slopes you computed in parts (a) and (b)?
- Why do you think  $m = \frac{(p_2 - r_2)}{(p_1 - r_1)} = \frac{(r_2 - p_2)}{(r_1 - p_1)}$ ?



10. Each of the lines in the lesson was non-vertical. Consider the slope of a vertical line,  $x = 2$ . Select two points on the line to calculate slope. Based on your answer, why do you think the topic of slope focuses only on non-vertical lines?



Challenge:

11. A certain line has a slope of  $\frac{1}{2}$ . Name two points that may be on the line.



3. Find at least three solutions to the equation  $y = 3x + 1$ , and graph the solutions as points on the coordinate plane. Connect the points to make a line. Find the slope of the line.
  
  
  
  
  
  
  
  
  
  
4. The graph of the equation  $y = 7x - 3$  has what slope?
  
  
  
  
  
5. The graph of the equation  $y = -\frac{3}{4}x - 3$  has what slope?
  
  
  
  
  
6. You have \$20 in savings at the bank. Each week, you add \$2 to your savings. Let  $y$  represent the total amount of money you have saved at the end of  $x$  weeks. Write an equation to represent this situation, and identify the slope of the equation. What does that number represent?
  
  
  
  
  
  
  
  
  
  
7. A friend is training for a marathon. She can run 4 miles in 28 minutes. Assume she runs at a constant rate. Write an equation to represent the total distance,  $y$ , your friend can run in  $x$  minutes. Identify the slope of the equation. What does that number represent?

8. Four boxes of pencils cost \$5. Write an equation that represents the total cost,  $y$ , for  $x$  boxes of pencils. What is the slope of the equation? What does that number represent?
9. Solve the following equation for  $y$ , and then identify the slope of the line:  $9x - 3y = 15$ .
10. Solve the following equation for  $y$ , and then identify the slope of the line:  $5x + 9y = 8$ .
11. Solve the following equation for  $y$ , and then identify the slope of the line:  $ax + by = c$ .

**Lesson Summary**

The line joining two distinct points of the graph of the linear equation  $y = mx + b$  has slope  $m$ .

The  $m$  of  $y = mx + b$  is the number that describes the slope. For example, in the equation  $y = -2x + 4$ , the slope of the graph of the line is  $-2$ .

**Problem Set**

1. Solve the following equation for  $y$ :  $-4x + 8y = 24$ . Then, answer the questions that follow.
  - a. Based on your transformed equation, what is the slope of the linear equation  $-4x + 8y = 24$ ?
  - b. Complete the table to find solutions to the linear equation.

$x$	Transformed Linear Equation:	$y$

- c. Graph the points on the coordinate plane.
- d. Find the slope between any two points.
- e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form  $y = mx + b$  that has slope  $m$ .
- f. Note the location (ordered pair) that describes where the line intersects the  $y$ -axis.

2. Solve the following equation for  $y$ :  $9x + 3y = 21$ . Then, answer the questions that follow.
- Based on your transformed equation, what is the slope of the linear equation  $9x + 3y = 21$ ?
  - Complete the table to find solutions to the linear equation.

$x$	Transformed Linear Equation:	$y$

- Graph the points on the coordinate plane.
  - Find the slope between any two points.
  - The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form  $y = mx + b$  that has slope  $m$ .
  - Note the location (ordered pair) that describes where the line intersects the  $y$ -axis.
3. Solve the following equation for  $y$ :  $2x + 3y = -6$ . Then, answer the questions that follow.
- Based on your transformed equation, what is the slope of the linear equation  $2x + 3y = -6$ ?
  - Complete the table to find solutions to the linear equation.

$x$	Transformed Linear Equation:	$y$

- Graph the points on the coordinate plane.
- Find the slope between any two points.
- The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form  $y = mx + b$  that has slope  $m$ .
- Note the location (ordered pair) that describes where the line intersects the  $y$ -axis.

4. Solve the following equation for  $y$ :  $5x - y = 4$ . Then, answer the questions that follow.
- Based on your transformed equation, what is the slope of the linear equation  $5x - y = 4$ ?
  - Complete the table to find solutions to the linear equation.

$x$	Transformed Linear Equation:	$y$

- Graph the points on the coordinate plane.
- Find the slope between any two points.
- The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form  $y = mx + b$  that has slope  $m$ .
- Note the location (ordered pair) that describes where the line intersects the  $y$ -axis.

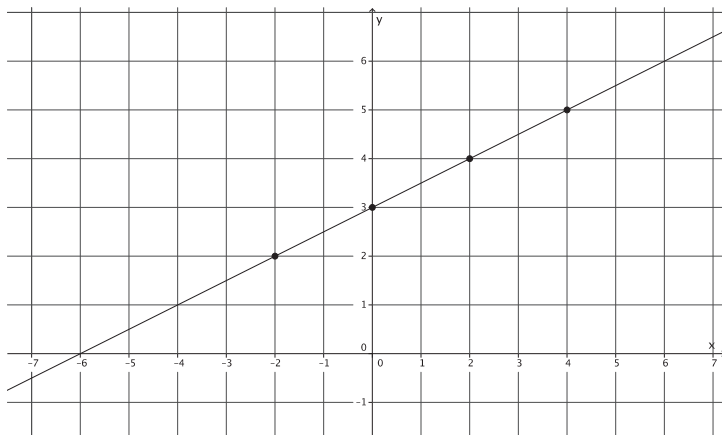
## Lesson 18: There Is Only One Line Passing Through a Given Point with a Given Slope

### Classwork

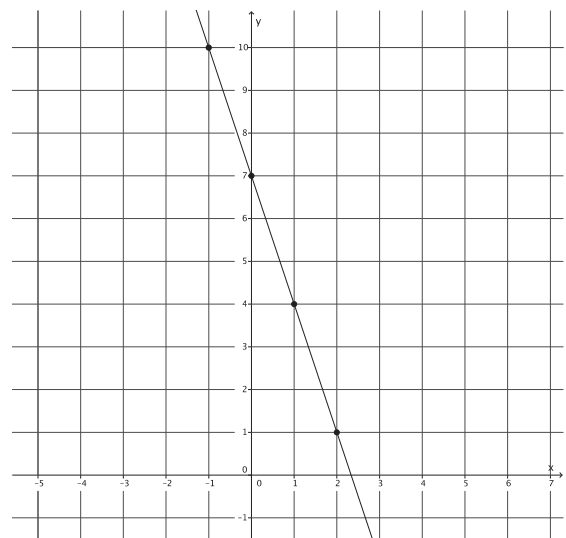
#### Opening Exercise

Examine each of the graphs and their equations. Identify the coordinates of the point where the line intersects the  $y$ -axis. Describe the relationship between the point and the equation  $y = mx + b$ .

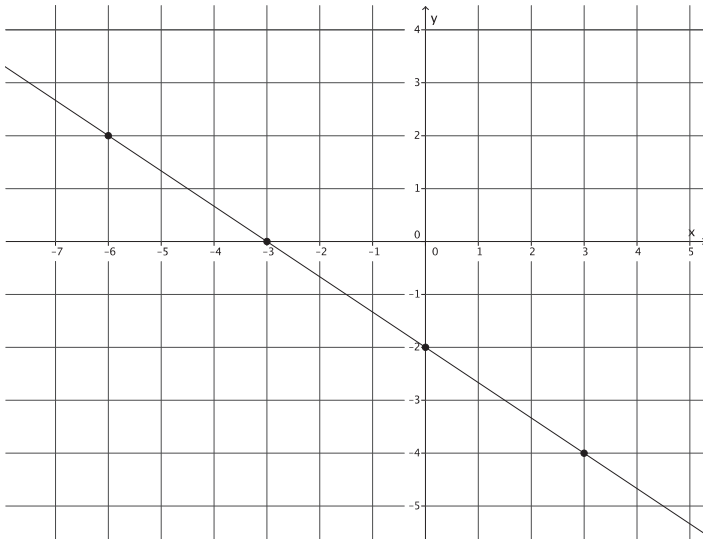
a.  $y = \frac{1}{2}x + 3$



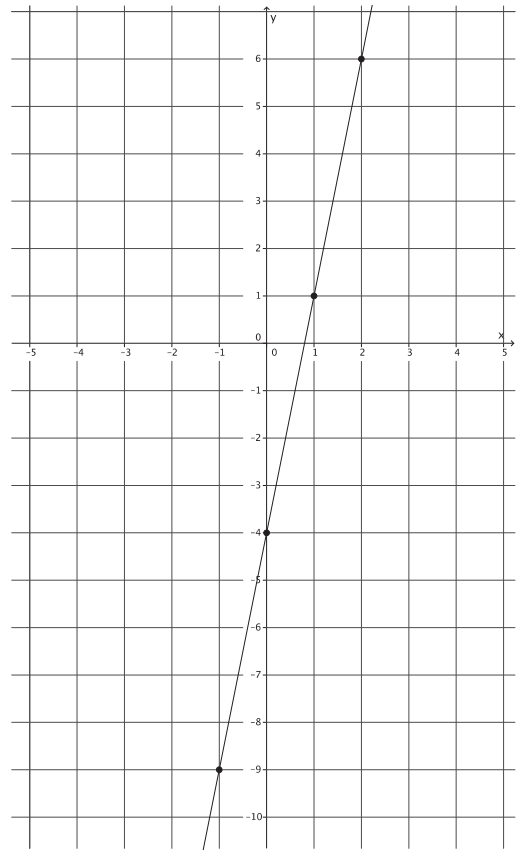
b.  $y = -3x + 7$



c.  $y = -\frac{2}{3}x - 2$

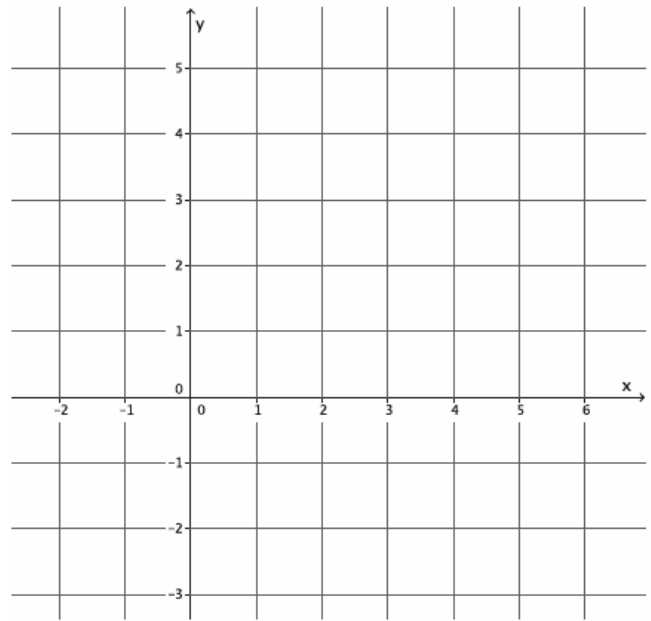


d.  $y = 5x - 4$

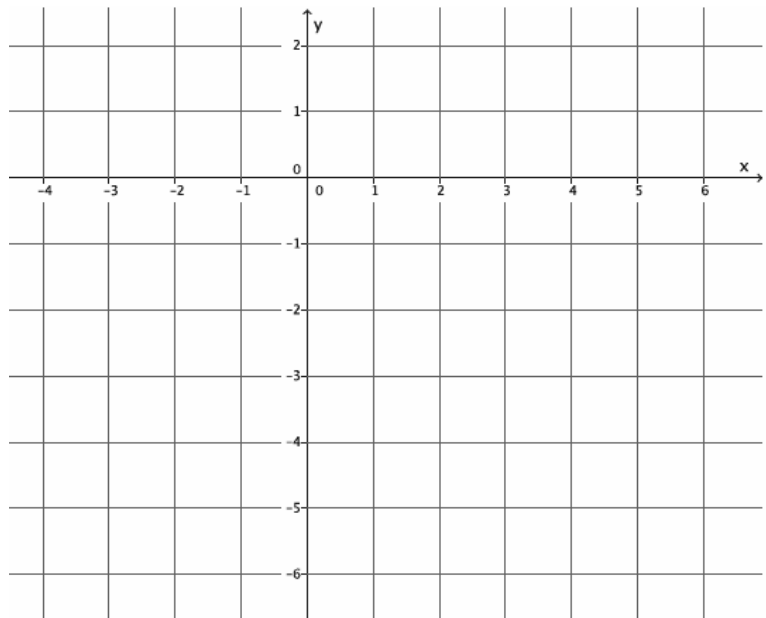


**Example 1**

Graph the equation  $y = \frac{2}{3}x + 1$ . Name the slope and  $y$ -intercept point.

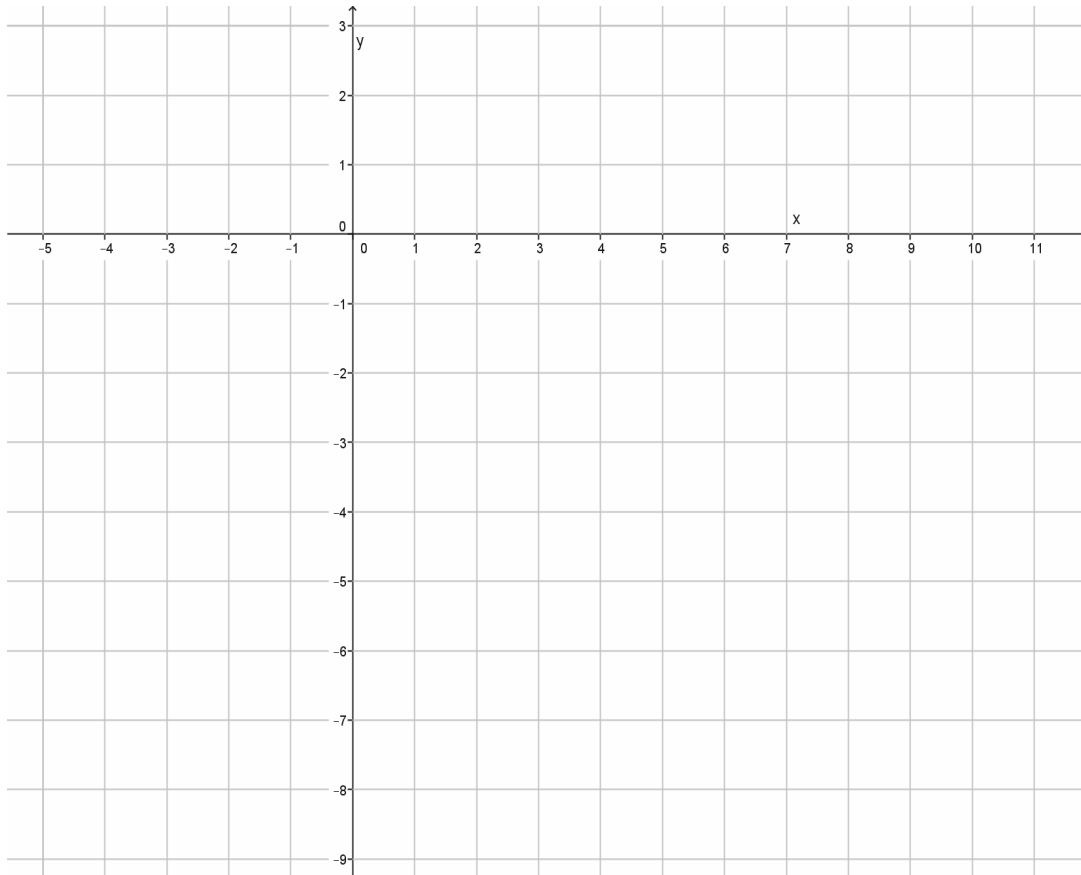
**Example 2**

Graph the equation  $y = -\frac{3}{4}x - 2$ . Name the slope and  $y$ -intercept point.



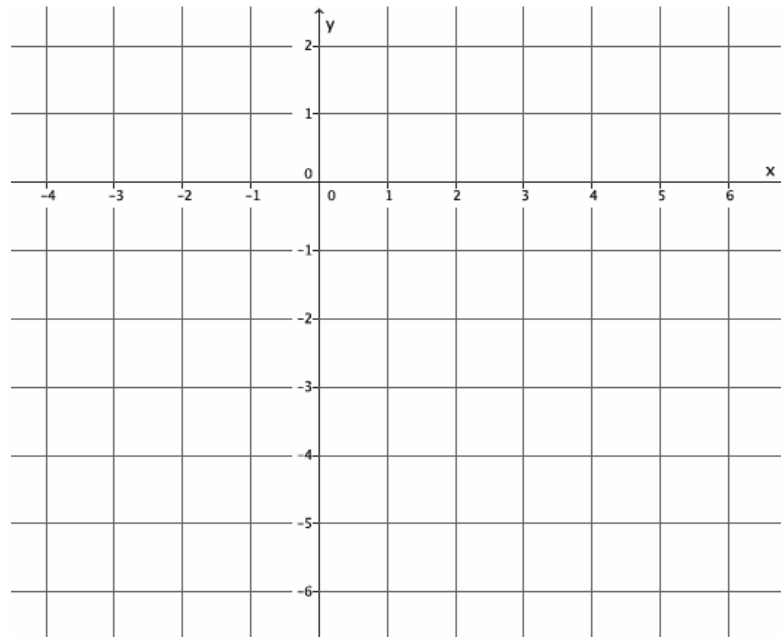
**Example 3**

Graph the equation  $y = 4x - 7$ . Name the slope and  $y$ -intercept point.

**Exercises**

- Graph the equation  $y = \frac{5}{2}x - 4$ .
  - Name the slope and the  $y$ -intercept point.

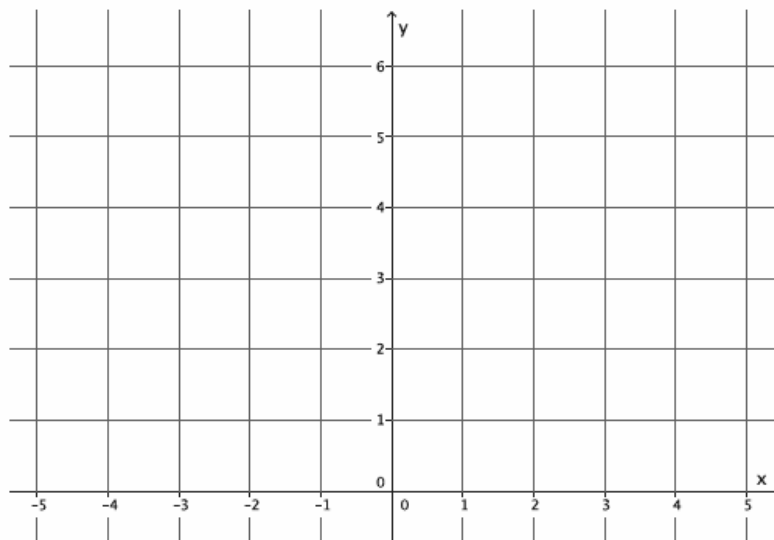
- b. Graph the known point, and then use the slope to find a second point before drawing the line.



2. Graph the equation  $y = -3x + 6$ .

- a. Name the slope and the  $y$ -intercept point.

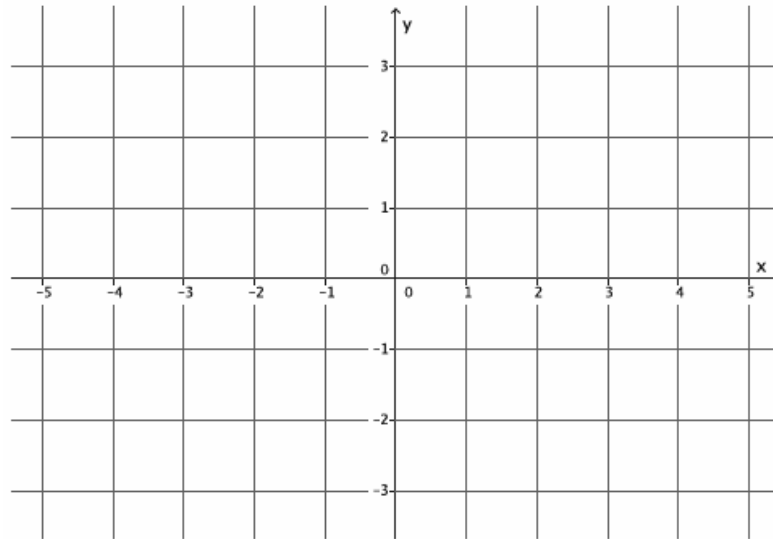
- b. Graph the known point, and then use the slope to find a second point before drawing the line.



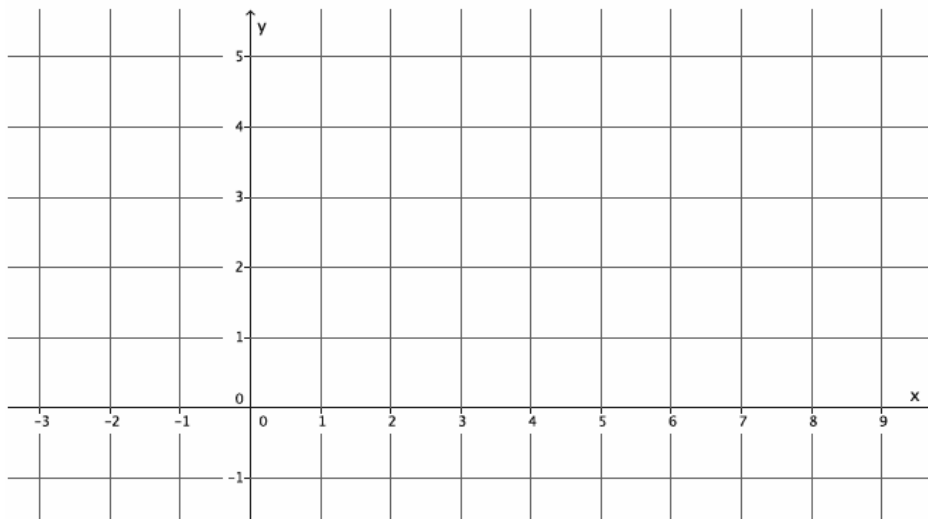
3. The equation  $y = 1x + 0$  can be simplified to  $y = x$ . Graph the equation  $y = x$ .

a. Name the slope and the  $y$ -intercept point.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



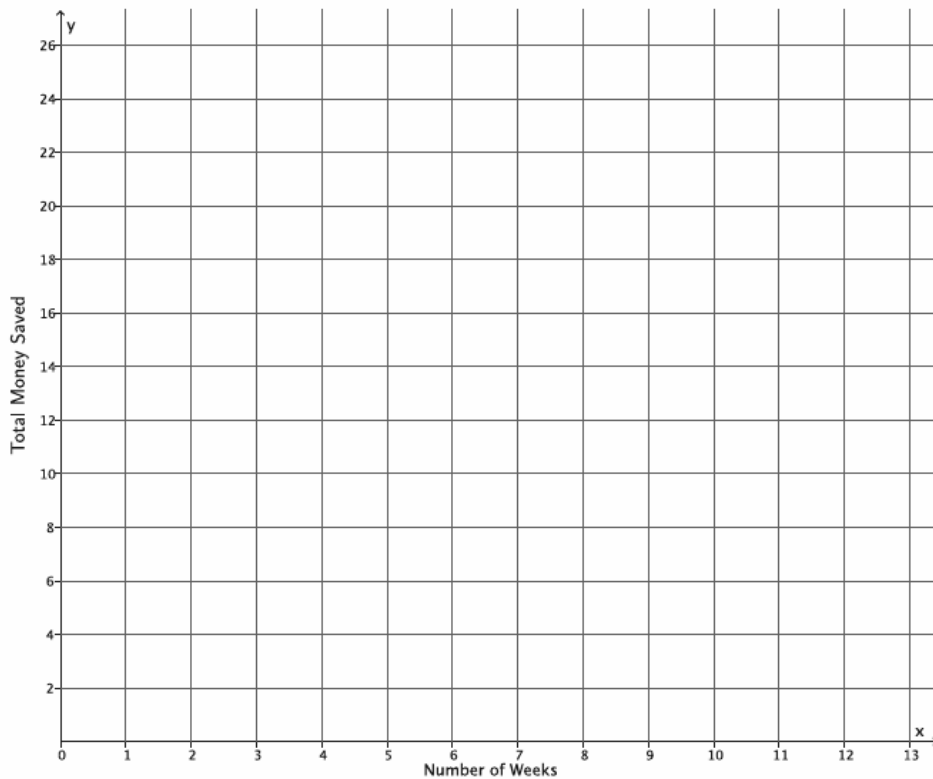
4. Graph the point  $(0, 2)$ .



a. Find another point on the graph using the slope,  $m = \frac{2}{7}$ .

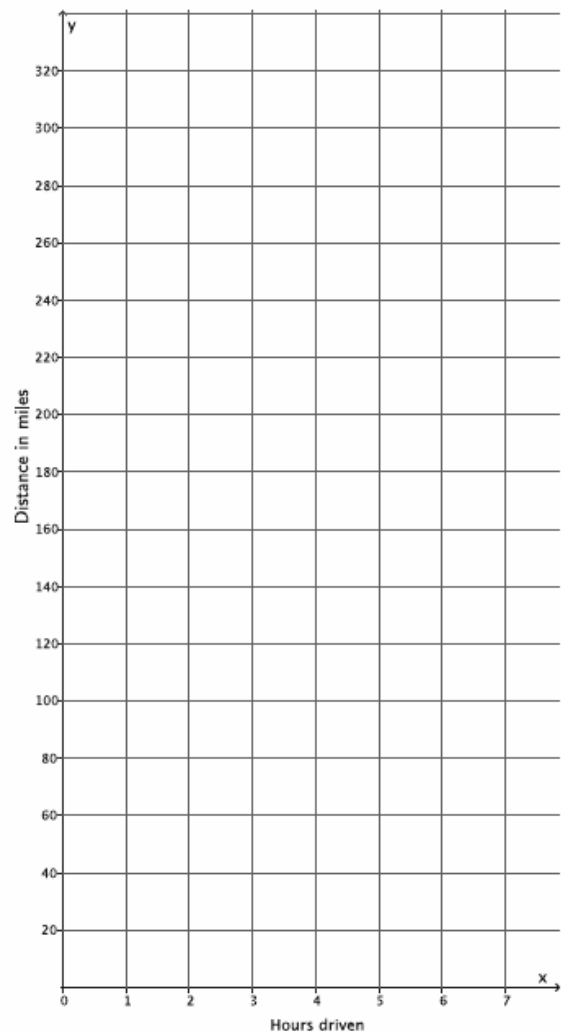
b. Connect the points to make the line.

- c. Draw a different line that goes through the point  $(0, 2)$  with slope  $m = \frac{2}{7}$ . What do you notice?
5. A bank put \$10 into a savings account when you opened the account. Eight weeks later, you have a total of \$24. Assume you saved the same amount every week.
- a. If  $y$  is the total amount of money in the savings account and  $x$  represents the number of weeks, write an equation in the form  $y = mx + b$  that describes the situation.
- b. Identify the slope and the  $y$ -intercept point. What do these numbers represent?
- c. Graph the equation on a coordinate plane.



- d. Could any other line represent this situation? For example, could a line through point  $(0,10)$  with slope  $\frac{7}{5}$  represent the amount of money you save each week? Explain.
6. A group of friends are on a road trip. After 120 miles, they stop to eat lunch. They continue their trip and drive at a constant rate of 50 miles per hour.
- a. Let  $y$  represent the total distance traveled, and let  $x$  represent the number of hours driven after lunch. Write an equation to represent the total number of miles driven that day.

- b. Identify the slope and the  $y$ -intercept point. What do these numbers represent?
- c. Graph the equation on a coordinate plane.
- d. Could any other line represent this situation? For example, could a line through point  $(0, 120)$  with slope 75 represent the total distance the friends drive? Explain.



**Lesson Summary**

The equation  $y = mx + b$  is in slope-intercept form. The number  $m$  represents the slope of the graph, and the point  $(0, b)$  is the location where the graph of the line intersects the  $y$ -axis.

To graph a line from the slope-intercept form of a linear equation, begin with the known point,  $(0, b)$ , and then use the slope to find a second point. Connect the points to graph the equation.

There is only one line passing through a given point with a given slope.

**Problem Set**

Graph each equation on a separate pair of  $x$ - and  $y$ -axes.

- Graph the equation  $y = \frac{4}{5}x - 5$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
- Graph the equation  $y = x + 3$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
- Graph the equation  $y = -\frac{4}{3}x + 4$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
- Graph the equation  $y = \frac{5}{2}x$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
- Graph the equation  $y = 2x - 6$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
- Graph the equation  $y = -5x + 9$ .
  - Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.

7. Graph the equation  $y = \frac{1}{3}x + 1$ .
- Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
8. Graph the equation  $5x + 4y = 8$ . (Hint: Transform the equation so that it is of the form  $y = mx + b$ .)
- Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
9. Graph the equation  $-2x + 5y = 30$ .
- Name the slope and the  $y$ -intercept point.
  - Graph the known point, and then use the slope to find a second point before drawing the line.
10. Let  $l$  and  $l'$  be two lines with the same slope  $m$  passing through the same point  $P$ . Show that there is only one line with a slope  $m$ , where  $m < 0$ , passing through the given point  $P$ . Draw a diagram if needed.

## Lesson 19: The Graph of a Linear Equation in Two Variables Is a Line

### Classwork

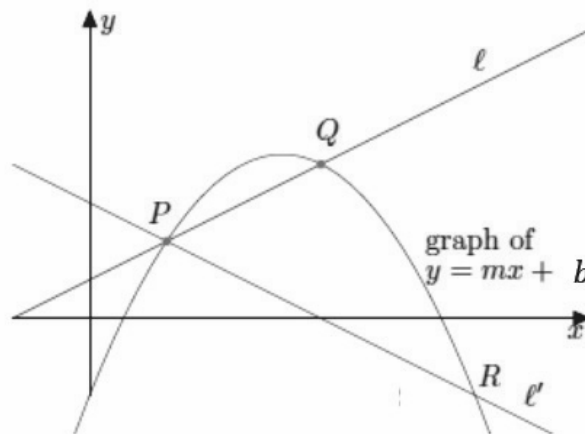
#### Exercises

**THEOREM:** The graph of a linear equation  $y = mx + b$  is a non-vertical line with slope  $m$  and passing through  $(0, b)$ , where  $b$  is a constant.

1. Prove the theorem by completing parts (a)–(c). Given two distinct points,  $P$  and  $Q$ , on the graph of  $y = mx + b$ , and let  $l$  be the line passing through  $P$  and  $Q$ . You must show the following:

- (1) Any point on the graph of  $y = mx + b$  is on line  $l$ , and
- (2) Any point on the line  $l$  is on the graph of  $y = mx + b$ .

- a. Proof of (1): Let  $R$  be any point on the graph of  $y = mx + b$ . Show that  $R$  is on  $l$ . Begin by assuming it is not. Assume the graph looks like the diagram below where  $R$  is on  $l'$ .

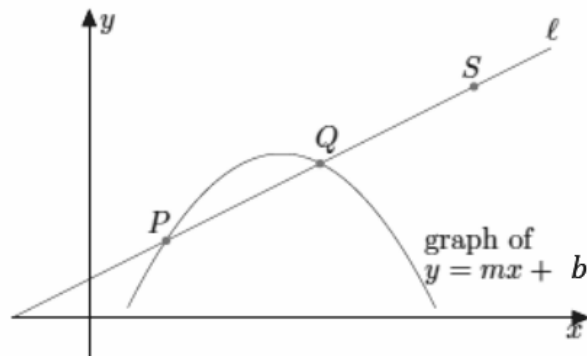


What is the slope of line  $l$ ?

What is the slope of line  $l'$ ?

What can you conclude about lines  $l$  and  $l'$ ? Explain.

- b. Proof of (2): Let  $S$  be any point on line  $l$ , as shown.



Show that  $S$  is a solution to  $y = mx + b$ . Hint: Use the point  $(0, b)$ .

- c. Now that you have shown that any point on the graph of  $y = mx + b$  is on line  $l$  in part (a), and any point on line  $l$  is on the graph of  $y = mx + b$  in part (b), what can you conclude about the graphs of linear equations?
2. Use  $x = 4$  and  $x = -4$  to find two solutions to the equation  $x + 2y = 6$ . Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation  $x + 2y = 6$ .
- b. When  $x = 1$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
- c. When  $x = -3$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
- d. Is the point  $(3, 2)$  on the line?
- e. Is the point  $(3, 2)$  a solution to the linear equation  $x + 2y = 6$ ?

3. Use  $x = 4$  and  $x = 1$  to find two solutions to the equation  $3x - y = 9$ . Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation  $3x - y = 9$ .
  - When  $x = 4.5$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
  - When  $x = \frac{1}{2}$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
  - Is the point  $(2, 4)$  on the line?
  - Is the point  $(2, 4)$  a solution to the linear equation  $3x - y = 9$ ?
4. Use  $x = 3$  and  $x = -3$  to find two solutions to the equation  $2x + 3y = 12$ . Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation  $2x + 3y = 12$ .

- b. When  $x = 2$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
- c. When  $x = -2$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
- d. Is the point  $(8, -3)$  on the line?
- e. Is the point  $(8, -3)$  a solution to the linear equation  $2x + 3y = 12$ ?
5. Use  $x = 4$  and  $x = -4$  to find two solutions to the equation  $x - 2y = 8$ . Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation  $x - 2y = 8$ .
- b. When  $x = 7$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?

- c. When  $x = -3$ , what is the value of  $y$ ? Does this solution appear to be a point on the line?
- d. Is the point  $(-2, -3)$  on the line?
- e. Is the point  $(-2, -3)$  a solution to the linear equation  $x - 2y = 8$ ?
6. Based on your work in Exercises 2–5, what conclusions can you draw about the points on a line and solutions to a linear equation?
7. Based on your work in Exercises 2–5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.
8. Based on your work in Exercises 2–5, what conclusions can you draw about the graph of a linear equation?

9. Graph the equation  $-3x + 8y = 24$  using intercepts.

10. Graph the equation  $x - 6y = 15$  using intercepts.

11. Graph the equation  $4x + 3y = 21$  using intercepts.

**Lesson Summary**

The graph of a linear equation is a line. A linear equation can be graphed using two-points: the  $x$ -intercept point and the  $y$ -intercept point.

Example:

Graph the equation:  $2x + 3y = 9$ .

Replace  $x$  with zero, and solve for  $y$  to determine the  $y$ -intercept point.

$$2(0) + 3y = 9$$

$$3y = 9$$

$$y = 3$$

The  $y$ -intercept point is at  $(0, 3)$ .

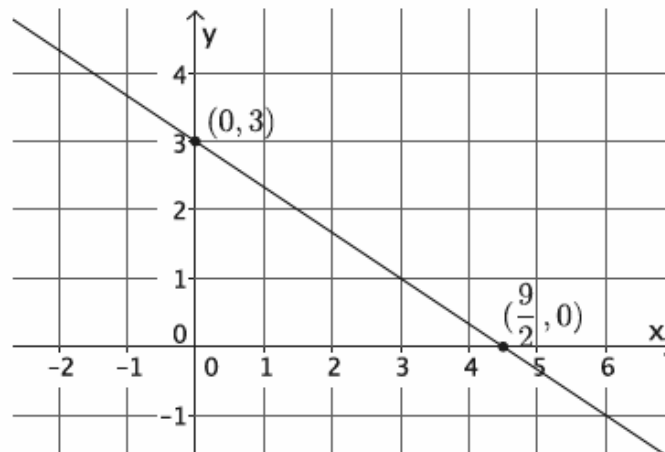
Replace  $y$  with zero, and solve for  $x$  to determine the  $x$ -intercept point.

$$2x + 3(0) = 9$$

$$2x = 9$$

$$x = \frac{9}{2}$$

The  $x$ -intercept point is at  $(\frac{9}{2}, 0)$ .



**Problem Set**

Graph each of the equations in the Problem Set on a different pair of  $x$ - and  $y$ -axes.

1. Graph the equation:  $y = -6x + 12$ .
2. Graph the equation:  $9x + 3y = 18$ .
3. Graph the equation:  $y = 4x + 2$ .
4. Graph the equation:  $y = -\frac{5}{7}x + 4$ .
5. Graph the equation:  $\frac{3}{4}x + y = 8$ .
6. Graph the equation:  $2x - 4y = 12$ .
7. Graph the equation:  $y = 3$ . What is the slope of the graph of this line?
8. Graph the equation:  $x = -4$ . What is the slope of the graph of this line?
9. Is the graph of  $4x + 5y = \frac{3}{7}$  a line? Explain.
10. Is the graph of  $6x^2 - 2y = 7$  a line? Explain.

## Lesson 20: Every Line Is a Graph of a Linear Equation

### Classwork

#### Opening Exercise

Figure 1

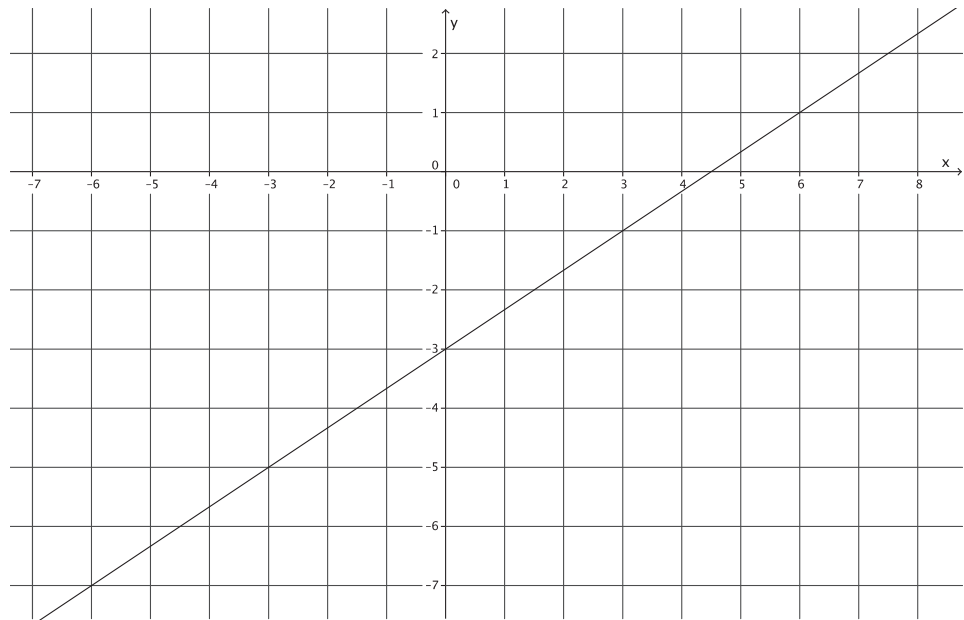
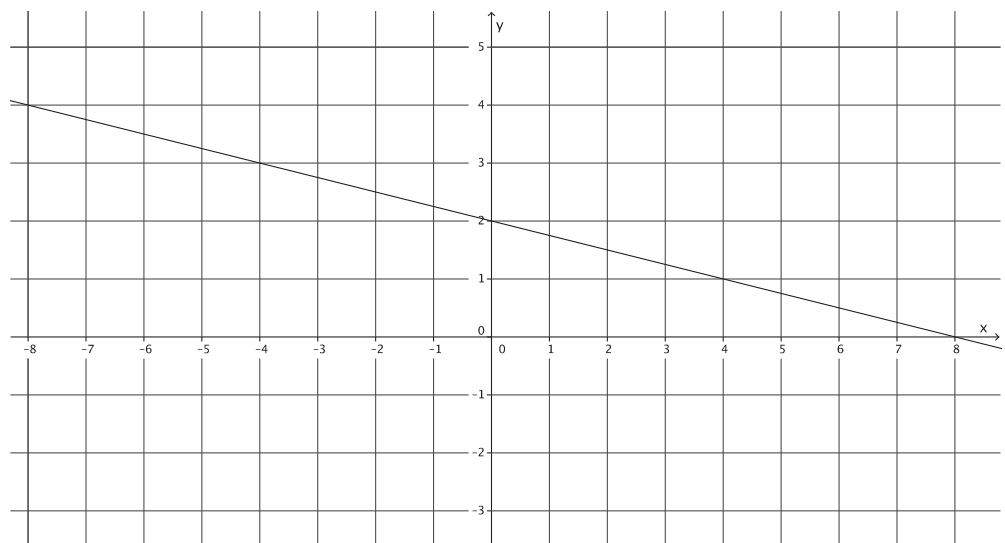


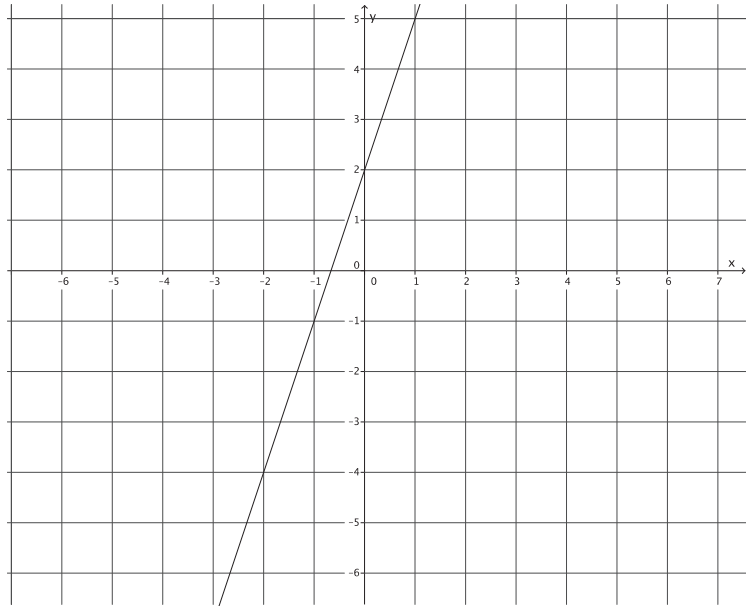
Figure 2



## Exercises

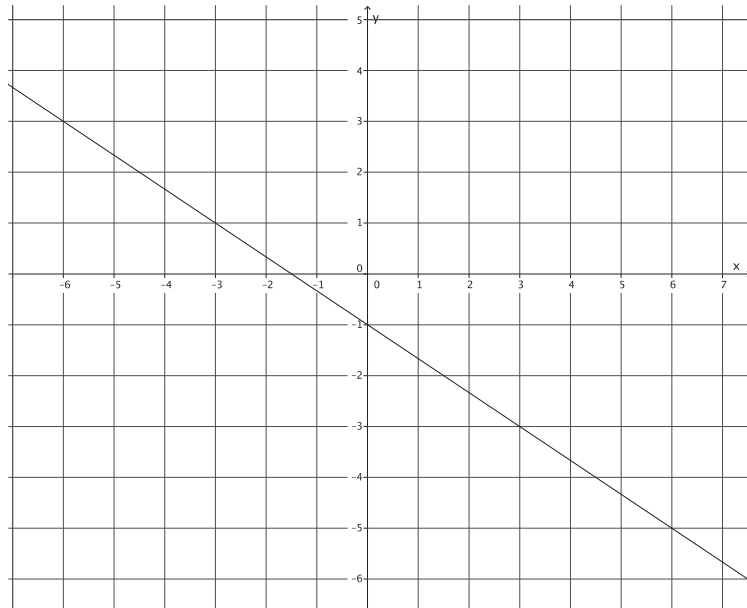
1. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



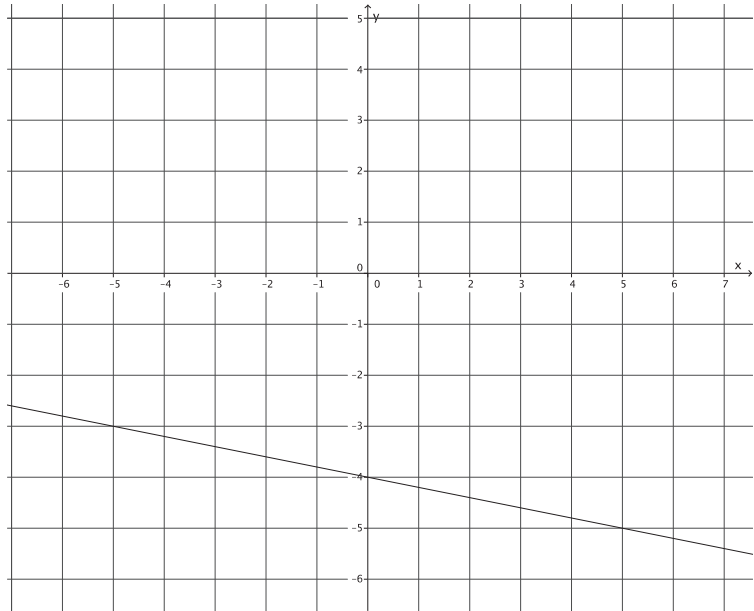
2. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



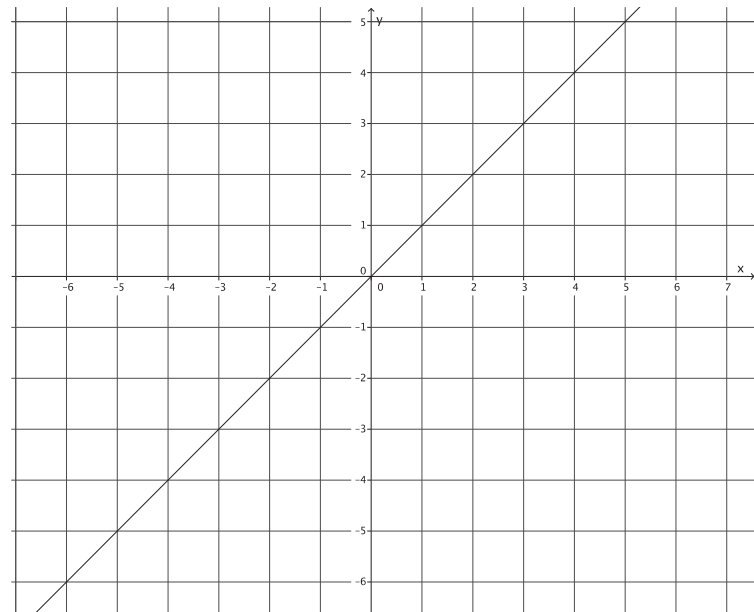
3. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



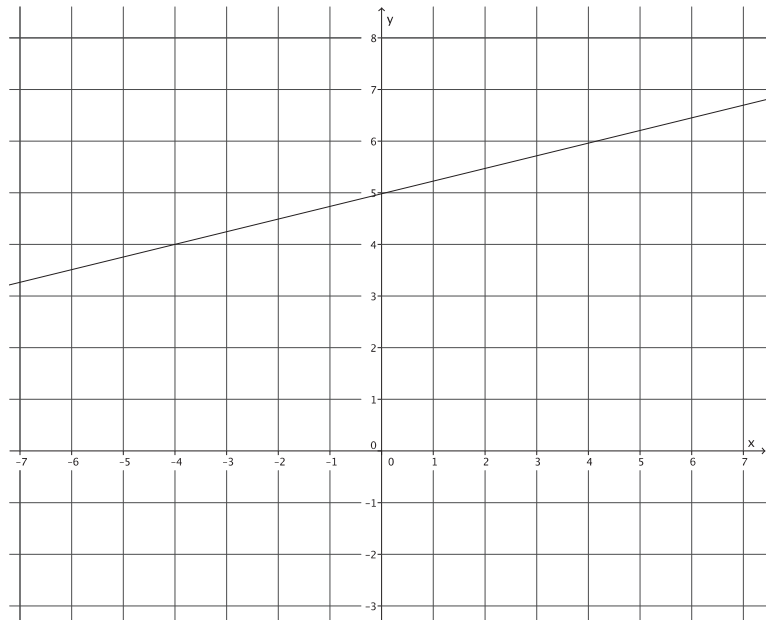
4. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



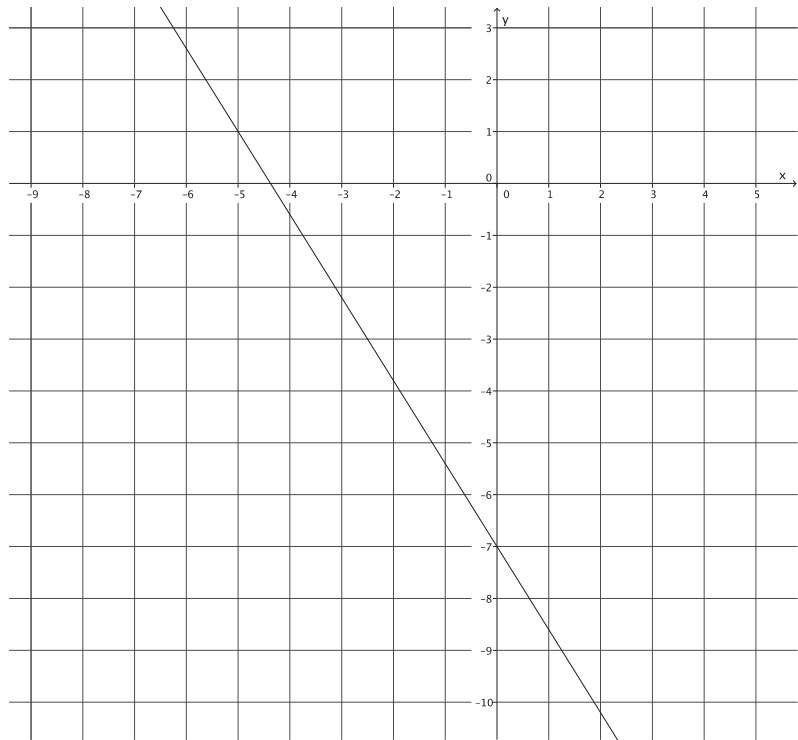
5. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



6. Write the equation that represents the line shown.

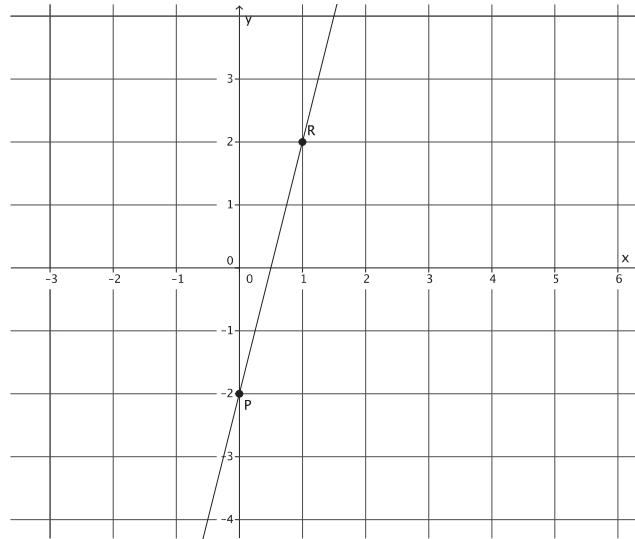
Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



**Lesson Summary**

Write the equation of a line by determining the  $y$ -intercept point,  $(0, b)$ , and the slope,  $m$ , and replacing the numbers  $b$  and  $m$  into the equation  $y = mx + b$ .

Example:



The  $y$ -intercept point of this graph is  $(0, -2)$ .

The slope of this graph is  $m = \frac{4}{1} = 4$ .

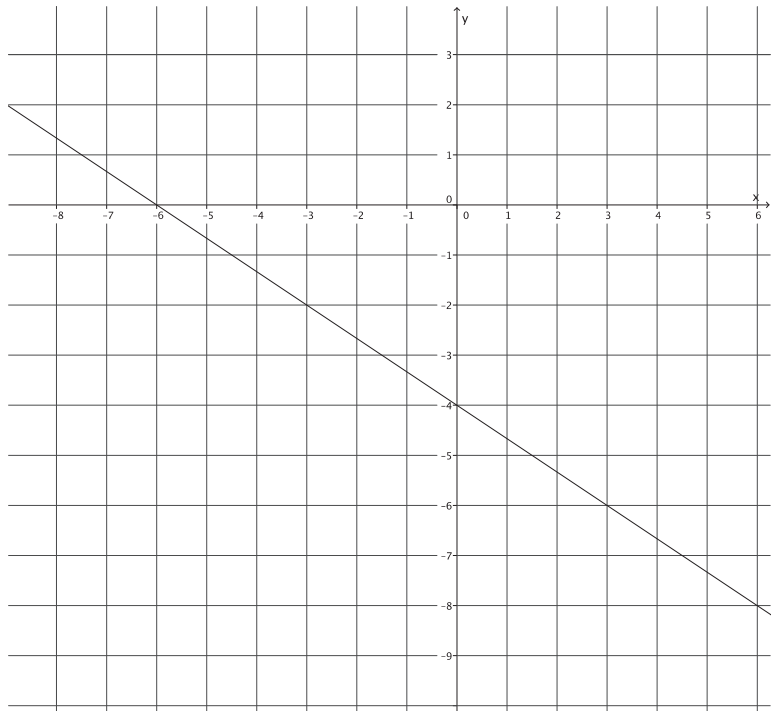
The equation that represents the graph of this line is  $y = 4x - 2$ .

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.

### Problem Set

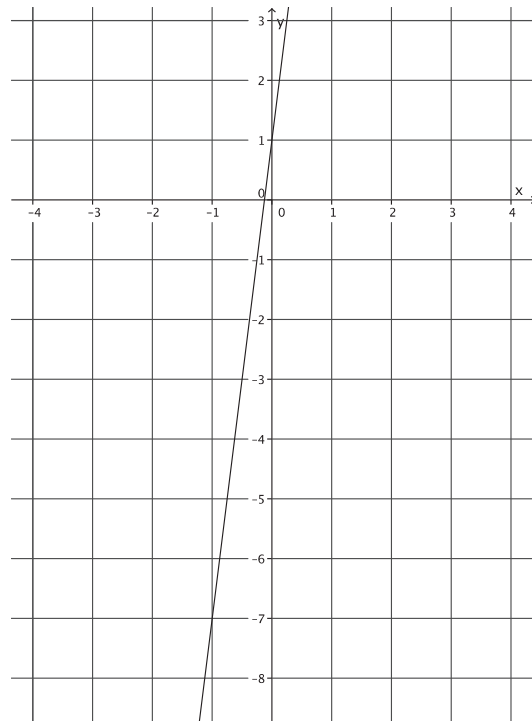
1. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



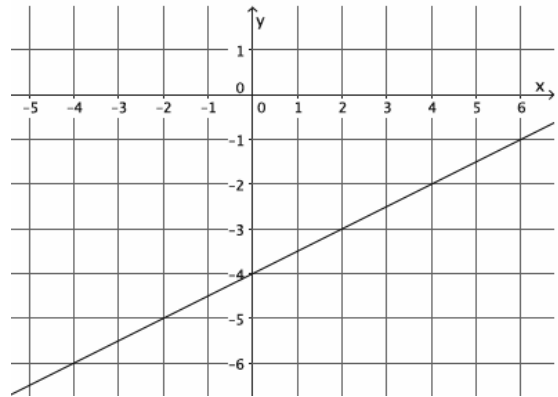
2. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



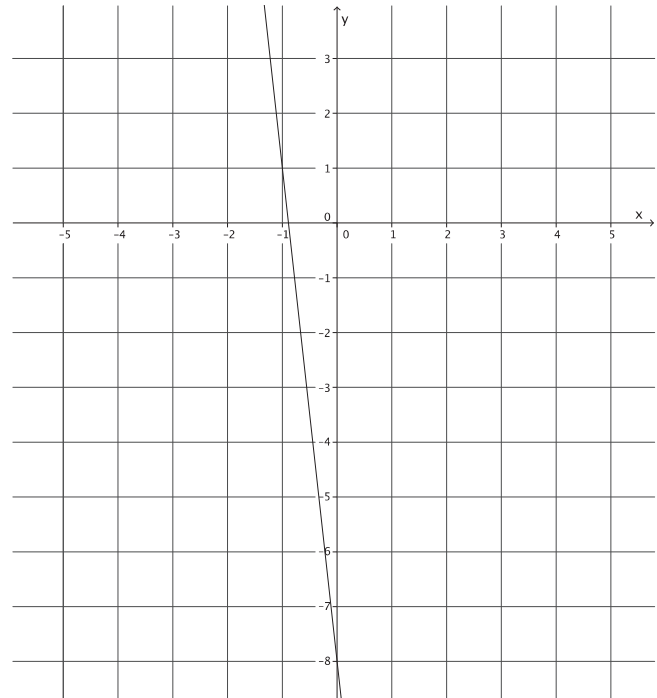
3. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



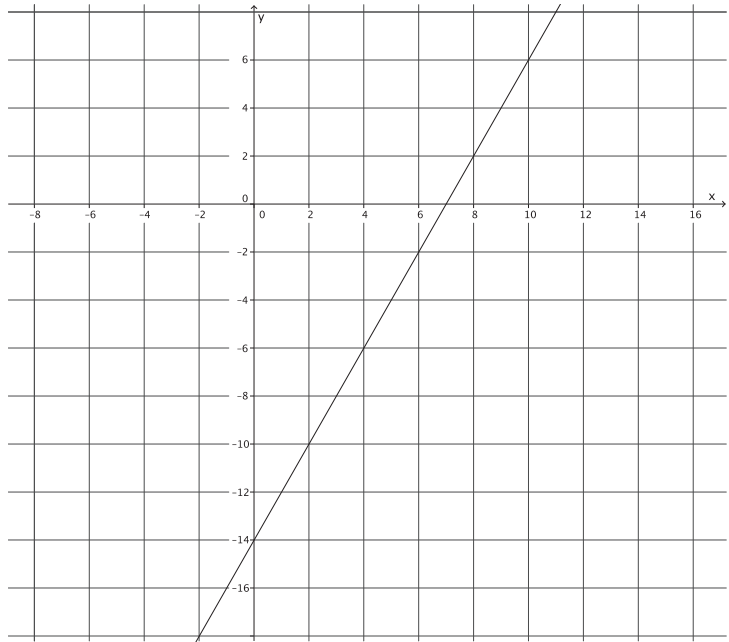
4. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



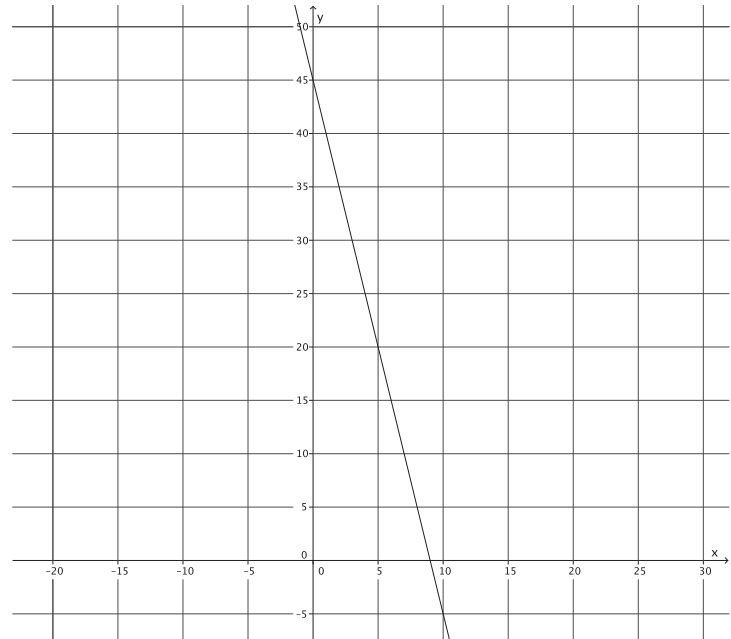
5. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.



6. Write the equation that represents the line shown.

Use the properties of equality to change the equation from slope-intercept form,  $y = mx + b$ , to standard form,  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are integers, and  $a$  is not negative.

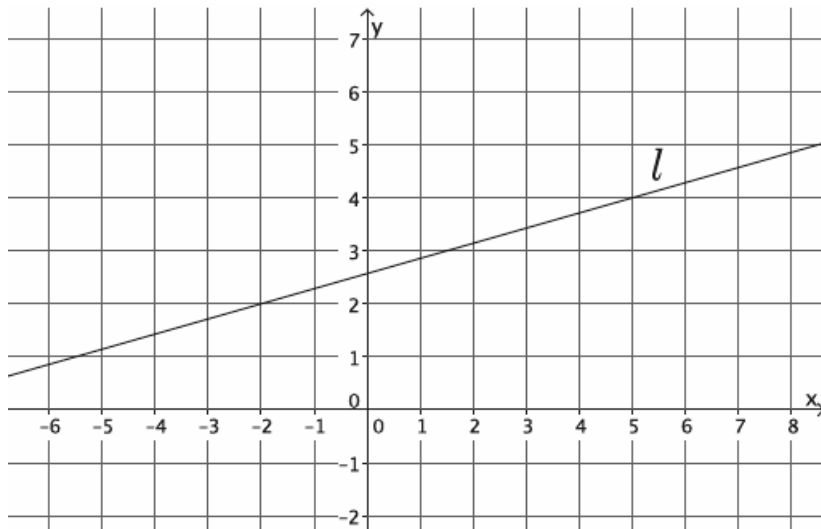


## Lesson 21: Some Facts About Graphs of Linear Equations in Two Variables

### Classwork

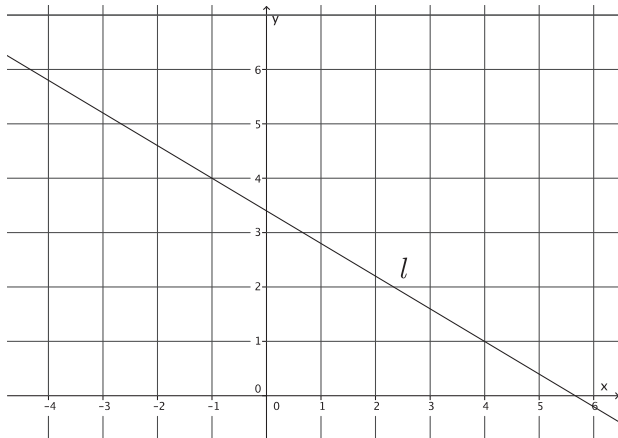
#### Example 1

Let a line  $l$  be given in the coordinate plane. What linear equation is the graph of line  $l$ ?



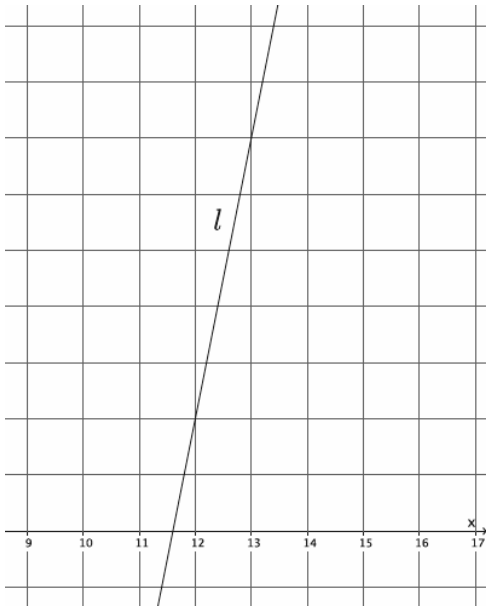
#### Example 2

Let a line  $l$  be given in the coordinate plane. What linear equation is the graph of line  $l$ ?

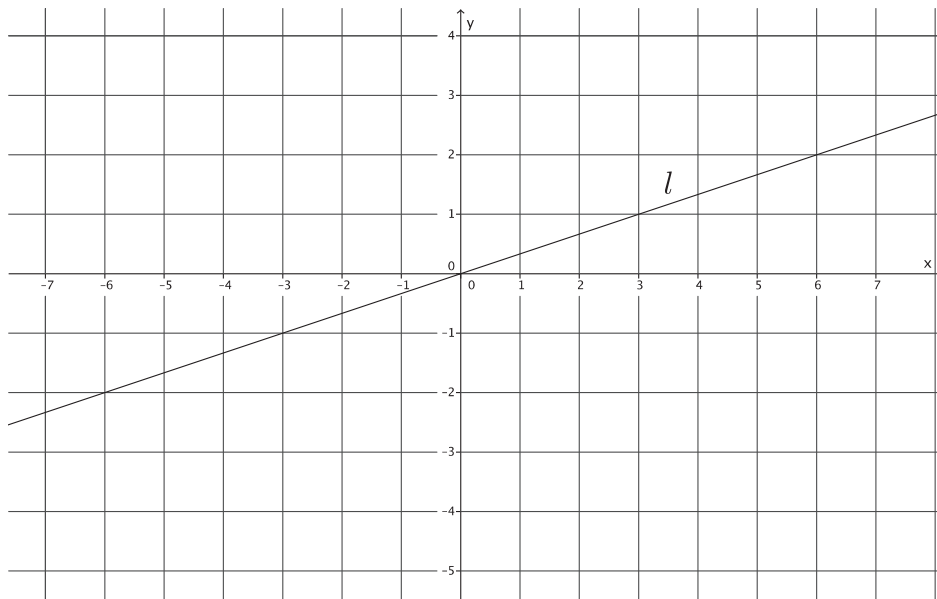


**Example 3**

Let a line  $l$  be given in the coordinate plane. What linear equation is the graph of line  $l$ ?

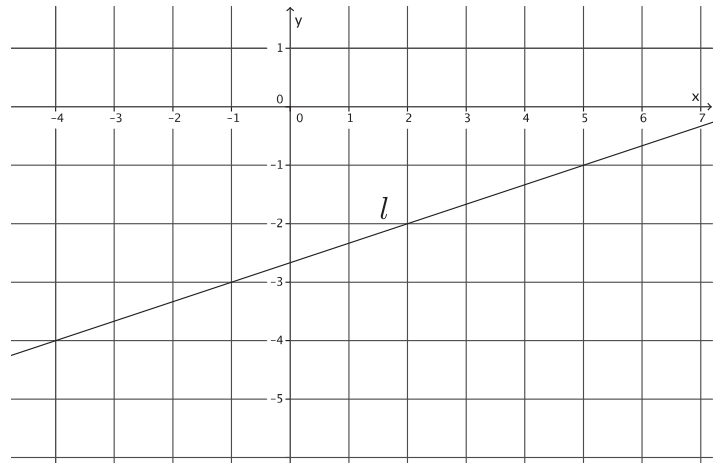
**Example 4**

Let a line  $l$  be given in the coordinate plane. What linear equation is the graph of line  $l$ ?

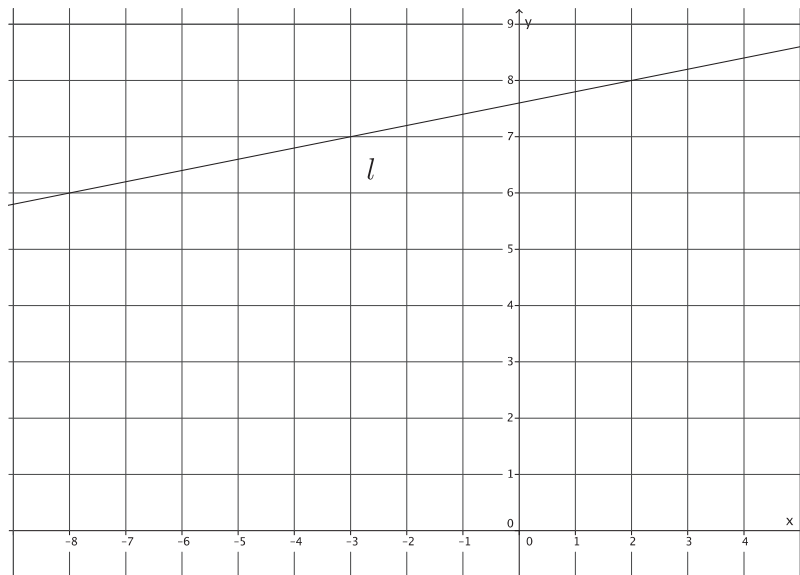


## Exercises

1. Write the equation for the line  $l$  shown in the figure.

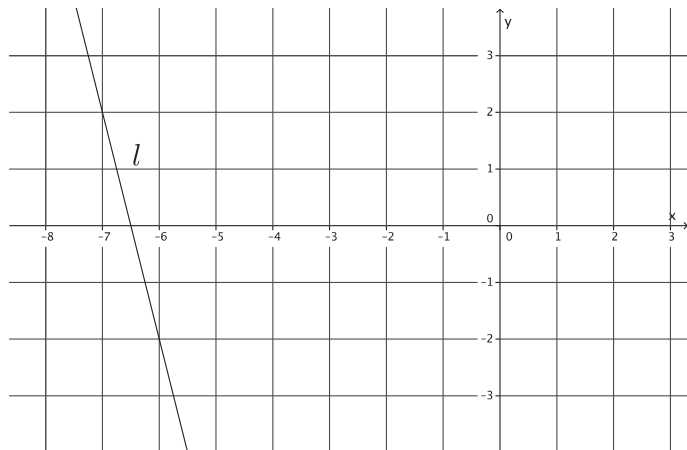


2. Write the equation for the line  $l$  shown in the figure.



3. Determine the equation of the line that goes through points  $(-4, 5)$  and  $(2, 3)$ .

4. Write the equation for the line  $l$  shown in the figure.



5. A line goes through the point  $(8, 3)$  and has slope  $m = 4$ . Write the equation that represents the line.

**Lesson Summary**

Let  $(x_1, y_1)$  and  $(x_2, y_2)$  be the coordinates of two distinct points on a non-vertical line in a coordinate plane. We find the slope of the line by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

This version of the slope formula, using coordinates of  $x$  and  $y$  instead of  $p$  and  $r$ , is a commonly accepted version.

As soon as you multiply the slope by the denominator of the fraction above, you get the following equation:

$$m(x_2 - x_1) = y_2 - y_1.$$

This form of an equation is referred to as the *point-slope form* of a linear equation.

Given a known  $(x, y)$ , then the equation is written as

$$m(x - x_1) = (y - y_1).$$

The following is slope-intercept form of a line:

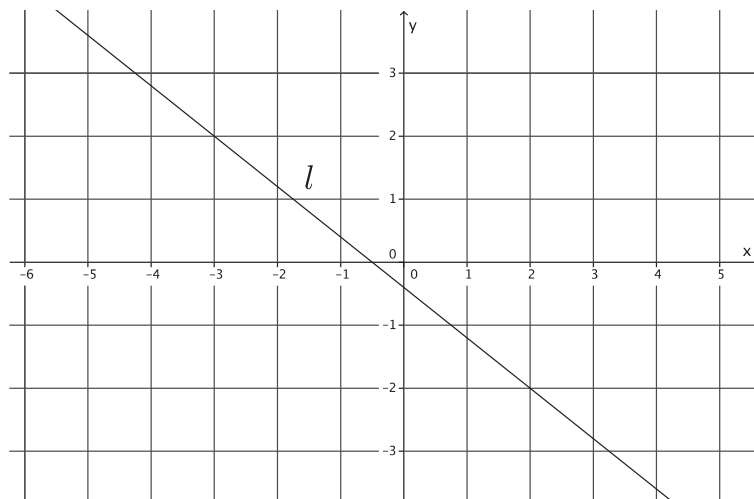
$$y = mx + b.$$

In this equation,  $m$  is slope, and  $(0, b)$  is the  $y$ -intercept point.

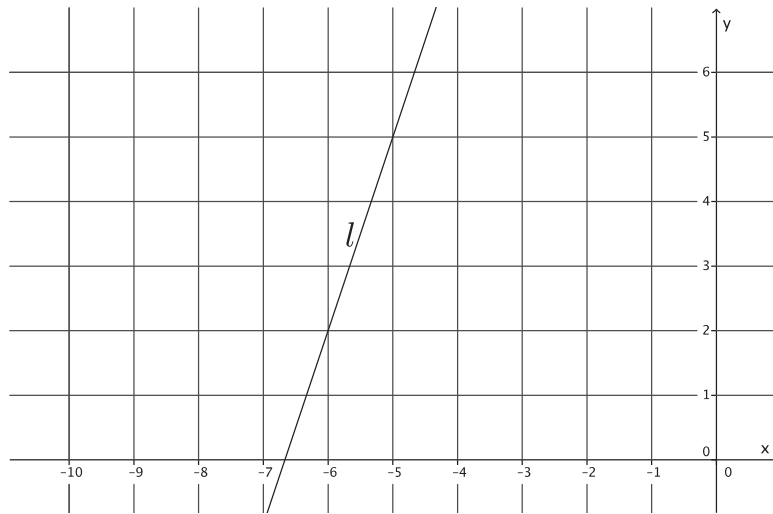
To write the equation of a line, you must have two points, one point and slope, or a graph of the line.

**Problem Set**

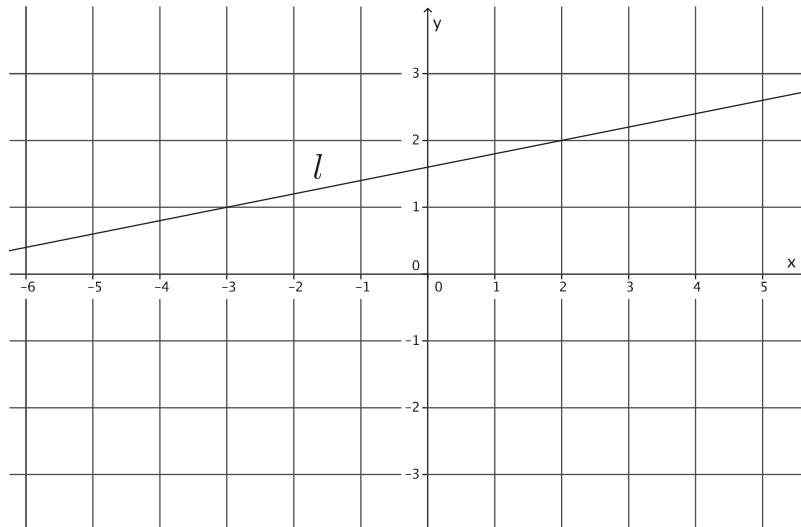
1. Write the equation for the line  $l$  shown in the figure.



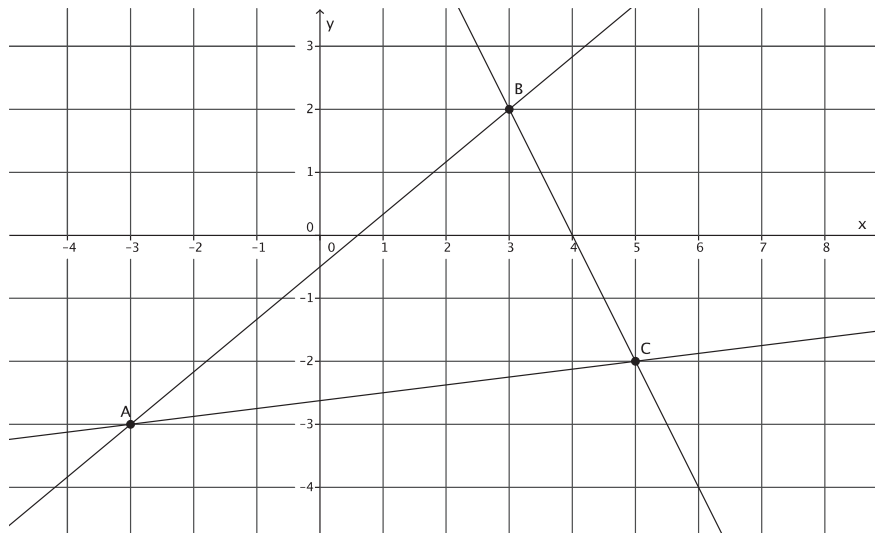
2. Write the equation for the line  $l$  shown in the figure.



3. Write the equation for the line  $l$  shown in the figure.



4. Triangle  $ABC$  is made up of line segments formed from the intersection of lines  $L_{AB}$ ,  $L_{BC}$ , and  $L_{AC}$ . Write the equations that represent the lines that make up the triangle.



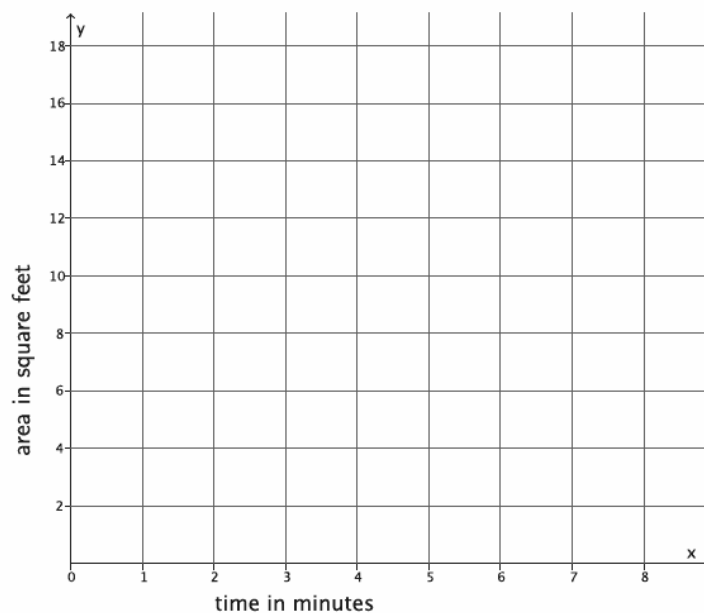
5. Write the equation for the line that goes through point  $(-10, 8)$  with slope  $m = 6$ .
6. Write the equation for the line that goes through point  $(12, 15)$  with slope  $m = -2$ .
7. Write the equation for the line that goes through point  $(1, 1)$  with slope  $m = -9$ .
8. Determine the equation of the line that goes through points  $(1, 1)$  and  $(3, 7)$ .

## Lesson 22: Constant Rates Revisited

### Classwork

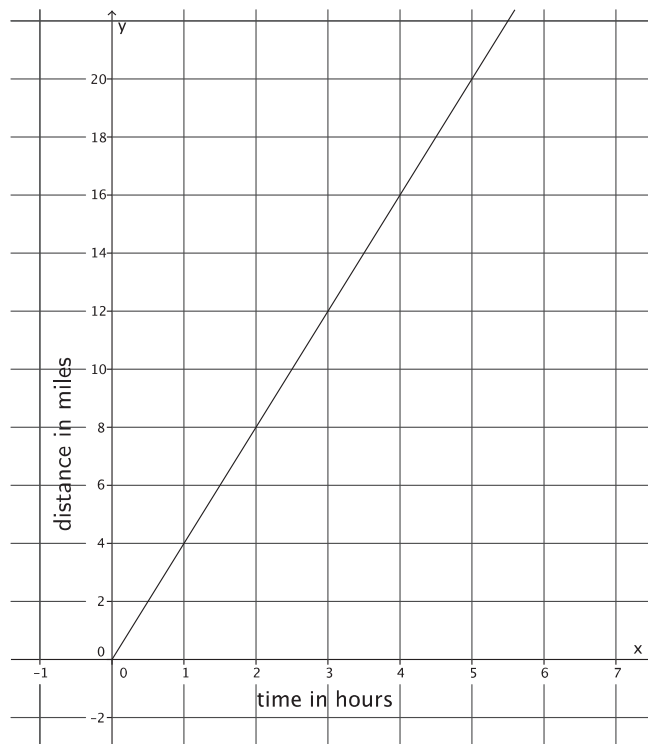
#### Exercises

- Peter paints a wall at a constant rate of 2 square feet per minute. Assume he paints an area  $y$ , in square feet, after  $x$  minutes.
  - Express this situation as a linear equation in two variables.
  
  
  
  
  
  
  
  
  
  
  - Sketch the graph of the linear equation.



- c. Using the graph or the equation, determine the total area he paints after 8 minutes,  $1\frac{1}{2}$  hours, and 2 hours. Note that the units are in minutes and hours.

2. The figure below represents Nathan's constant rate of walking.



- a. Nicole just finished a 5-mile walkathon. It took her 1.4 hours. Assume she walks at a constant rate. Let  $y$  represent the distance Nicole walks in  $x$  hours. Describe Nicole's walking at a constant rate as a linear equation in two variables.

b. Who walks at a greater speed? Explain.

3.

a. Susan can type 4 pages of text in 10 minutes. Assuming she types at a constant rate, write the linear equation that represents the situation.

b. The table of values below represents the number of pages that Anne can type,  $y$ , in a few selected  $x$  minutes. Assume she types at a constant rate.

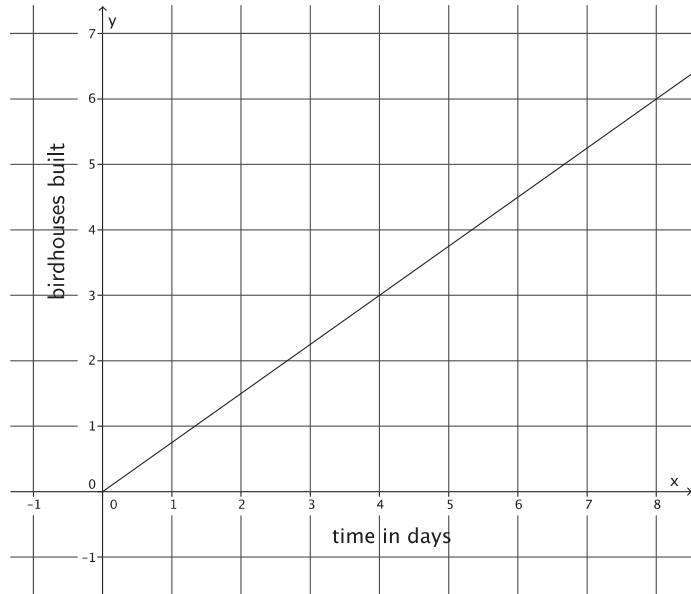
Minutes ( $x$ )	Pages Typed ( $y$ )
3	2
5	$\frac{10}{3}$
8	$\frac{16}{3}$
10	$\frac{20}{3}$

Who types faster? Explain.

4.

a. Phil can build 3 birdhouses in 5 days. Assuming he builds birdhouses at a constant rate, write the linear equation that represents the situation.

- b. The figure represents Karl's constant rate of building the same kind of birdhouses. Who builds birdhouses faster? Explain.



5. Explain your general strategy for comparing proportional relationships.

**Lesson Summary**

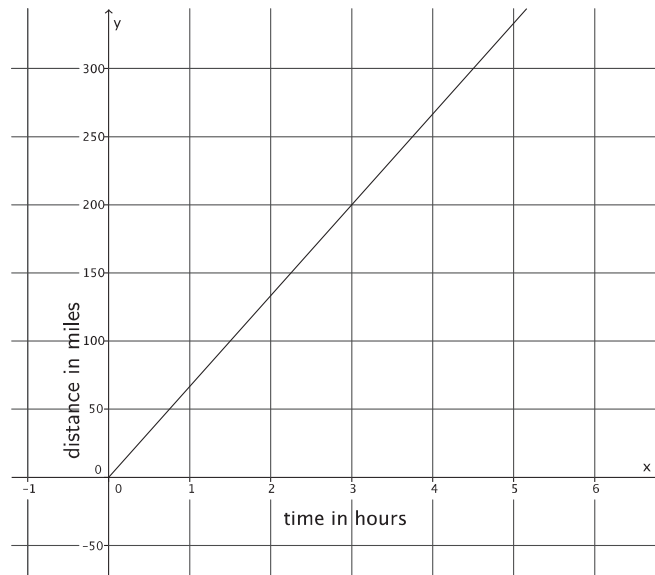
Problems involving constant rate can be expressed as linear equations in two variables.

When given information about two proportional relationships, their rates of change can be compared by comparing the slopes of the graphs of the two proportional relationships.

**Problem Set**

1.

- Train A can travel a distance of 500 miles in 8 hours. Assuming the train travels at a constant rate, write the linear equation that represents the situation.
- The figure represents the constant rate of travel for Train B.



Which train is faster? Explain.

2.

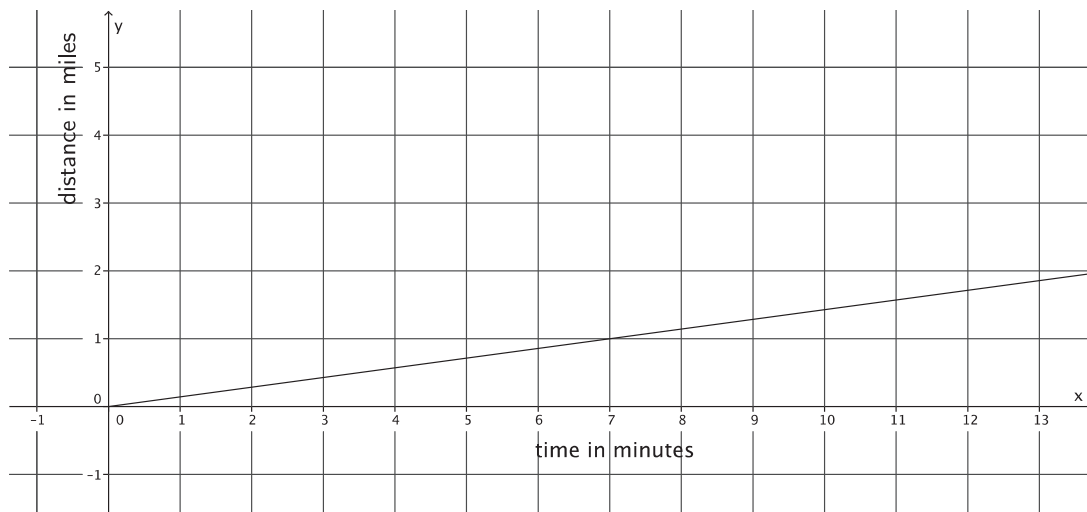
- a. Natalie can paint 40 square feet in 9 minutes. Assuming she paints at a constant rate, write the linear equation that represents the situation.
- b. The table of values below represents the area painted by Steven for a few selected time intervals. Assume Steven is painting at a constant rate.

Minutes ( $x$ )	Area Painted ( $y$ )
3	10
5	$\frac{50}{3}$
6	20
8	$\frac{80}{3}$

Who paints faster? Explain.

3.

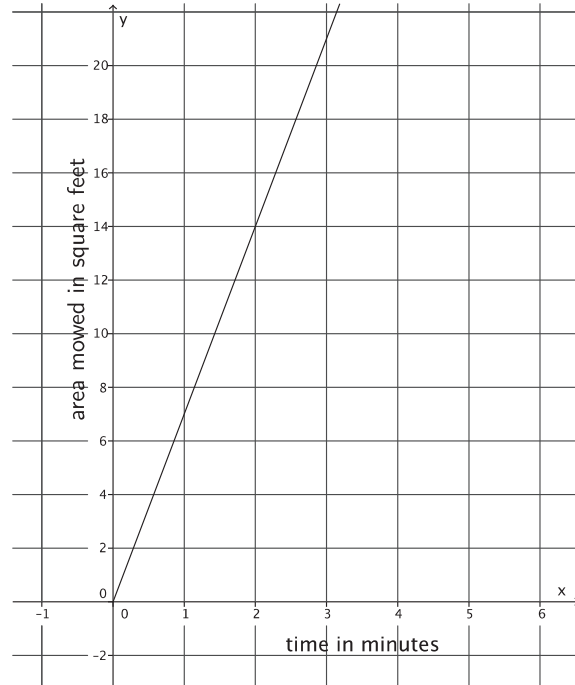
- a. Bianca can run 5 miles in 41 minutes. Assuming she runs at a constant rate, write the linear equation that represents the situation.
- b. The figure below represents Cynthia's constant rate of running.



Who runs faster? Explain.

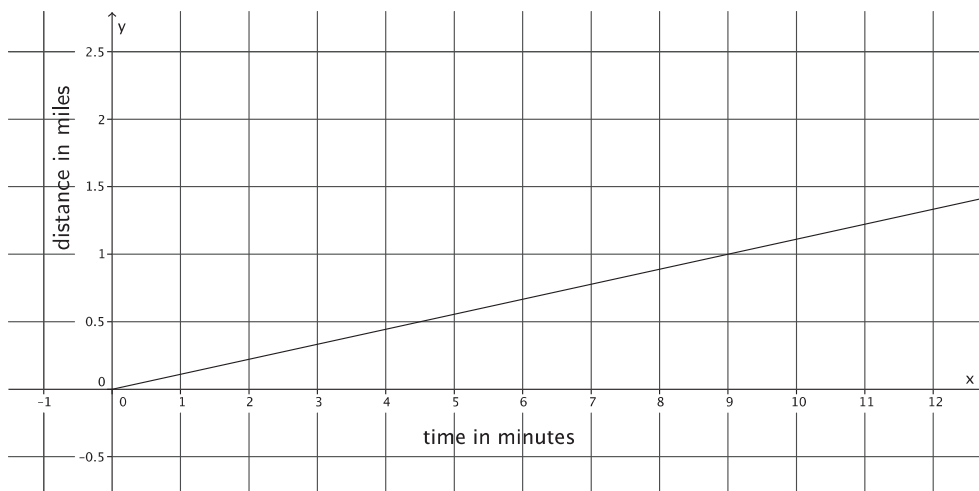
4.

- a. Geoff can mow an entire lawn of 450 square feet in 30 minutes. Assuming he mows at a constant rate, write the linear equation that represents the situation.
- b. The figure represents Mark's constant rate of mowing a lawn.  
Who mows faster? Explain.



5.

- a. Juan can walk to school, a distance of 0.75 mile, in 8 minutes. Assuming he walks at a constant rate, write the linear equation that represents the situation.
- b. The figure below represents Lena's constant rate of walking.



Who walks faster? Explain.

## Lesson 23: The Defining Equation of a Line

### Classwork

#### Exploratory Challenge/Exercises 1–3

1. Sketch the graph of the equation  $9x + 3y = 18$  using intercepts. Then, answer parts (a)–(f) that follow.
  - a. Sketch the graph of the equation  $y = -3x + 6$  on the same coordinate plane.
  - b. What do you notice about the graphs of  $9x + 3y = 18$  and  $y = -3x + 6$ ? Why do you think this is so?
  - c. Rewrite  $y = -3x + 6$  in standard form.
  - d. Identify the constants  $a$ ,  $b$ , and  $c$  of the equation in standard form from part (c).

- e. Identify the constants of the equation  $9x + 3y = 18$ . Note them as  $a'$ ,  $b'$ , and  $c'$ .
- f. What do you notice about  $\frac{a'}{a}$ ,  $\frac{b'}{b}$ , and  $\frac{c'}{c}$ ?
2. Sketch the graph of the equation  $y = \frac{1}{2}x + 3$  using the  $y$ -intercept point and the slope. Then, answer parts (a)–(f) that follow.
- a. Sketch the graph of the equation  $4x - 8y = -24$  using intercepts on the same coordinate plane.
- b. What do you notice about the graphs of  $y = \frac{1}{2}x + 3$  and  $4x - 8y = -24$ ? Why do you think this is so?
- c. Rewrite  $y = \frac{1}{2}x + 3$  in standard form.

- d. Identify the constants  $a$ ,  $b$ , and  $c$  of the equation in standard form from part (c).
- e. Identify the constants of the equation  $4x - 8y = -24$ . Note them as  $a'$ ,  $b'$ , and  $c'$ .
- f. What do you notice about  $\frac{a'}{a}$ ,  $\frac{b'}{b}$ , and  $\frac{c'}{c}$ ?

3. The graphs of the equations  $y = \frac{2}{3}x - 4$  and  $6x - 9y = 36$  are the same line.

- a. Rewrite  $y = \frac{2}{3}x - 4$  in standard form.

b. Identify the constants  $a$ ,  $b$ , and  $c$  of the equation in standard form from part (a).

c. Identify the constants of the equation  $6x - 9y = 36$ . Note them as  $a'$ ,  $b'$ , and  $c'$ .

- d. What do you notice about  $\frac{a'}{a}$ ,  $\frac{b'}{b}$ , and  $\frac{c'}{c}$ ?

- e. You should have noticed that each fraction was equal to the same constant. Multiply that constant by the standard form of the equation from part (a). What do you notice?

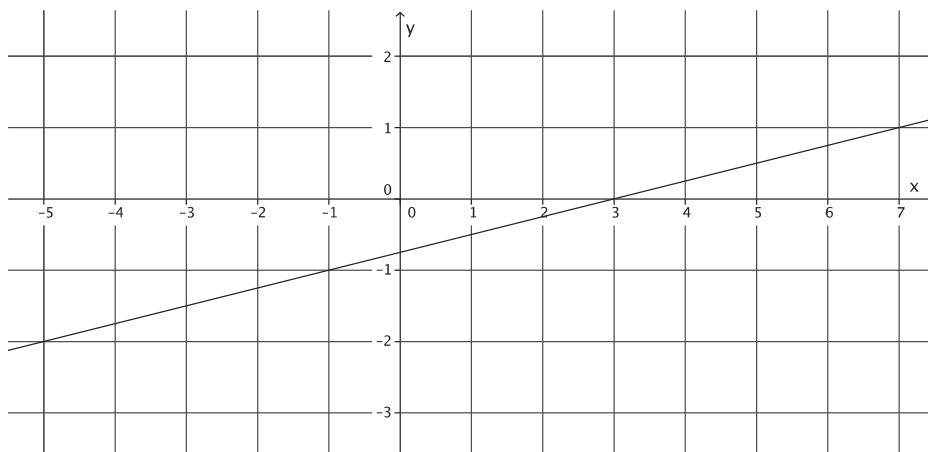
**Exercises 4–8**

4. Write three equations whose graphs are the same line as the equation  $3x + 2y = 7$ .

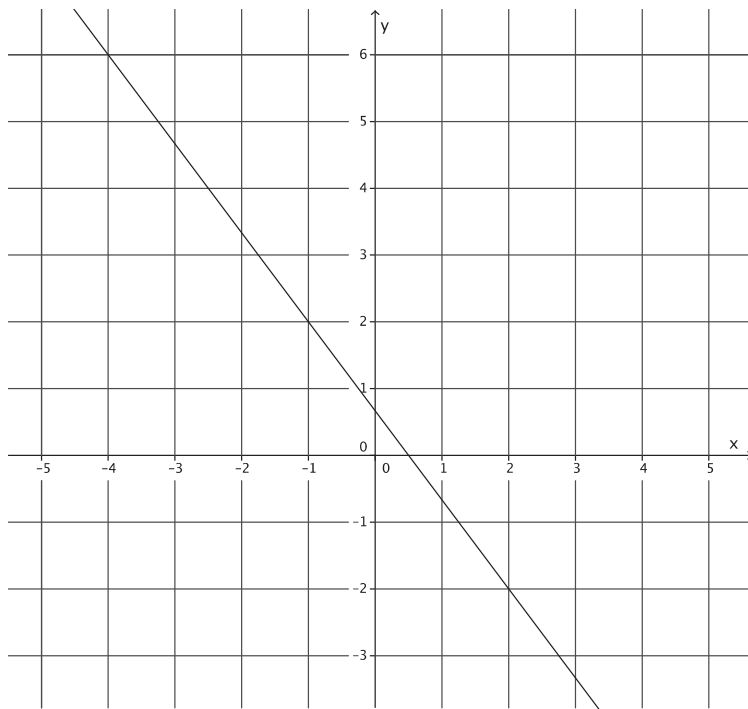
5. Write three equations whose graphs are the same line as the equation  $x - 9y = \frac{3}{4}$ .

6. Write three equations whose graphs are the same line as the equation  $-9x + 5y = -4$ .

7. Write at least two equations in the form  $ax + by = c$  whose graphs are the line shown below.



8. Write at least two equations in the form  $ax + by = c$  whose graphs are the line shown below.



**Lesson Summary**

Two equations define the same line if the graphs of those two equations are the same given line. Two equations that define the same line are the same equation, just in different forms. The equations may look different (different constants, different coefficients, or different forms).

When two equations are written in standard form,  $ax + by = c$  and  $a'x + b'y = c'$ , they define the same line when  $\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$  is true.

**Problem Set**

1. Do the equations  $x + y = -2$  and  $3x + 3y = -6$  define the same line? Explain.
2. Do the equations  $y = -\frac{5}{4}x + 2$  and  $10x + 8y = 16$  define the same line? Explain.
3. Write an equation that would define the same line as  $7x - 2y = 5$ .
4. Challenge: Show that if the two lines given by  $ax + by = c$  and  $a'x + b'y = c'$  are the same when  $b = 0$  (vertical lines), then there exists a nonzero number  $s$  so that  $a' = sa$ ,  $b' = sb$ , and  $c' = sc$ .
5. Challenge: Show that if the two lines given by  $ax + by = c$  and  $a'x + b'y = c'$  are the same when  $a = 0$  (horizontal lines), then there exists a nonzero number  $s$  so that  $a' = sa$ ,  $b' = sb$ , and  $c' = sc$ .

## Lesson 24: Introduction to Simultaneous Equations

### Classwork

#### Exercises

1. Derek scored 30 points in the basketball game he played, and not once did he go to the free throw line. That means that Derek scored two-point shots and three-point shots. List as many combinations of two- and three-pointers as you can that would total 30 points.

Number of Two-Pointers	Number of Three-Pointers

Write an equation to describe the data.

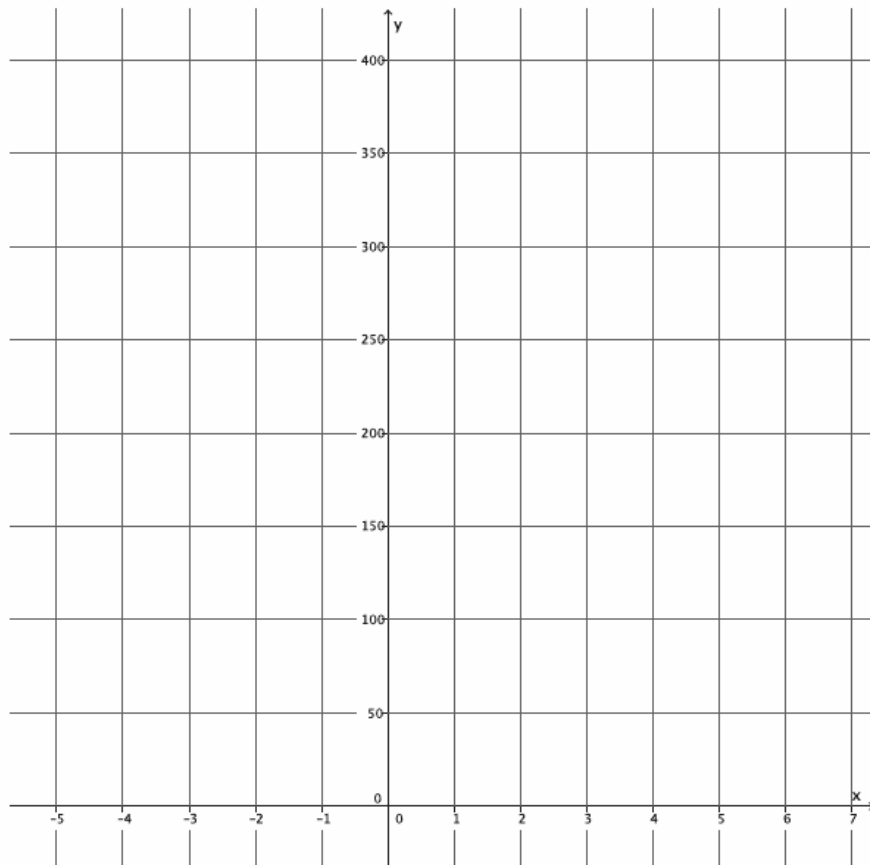
2. Derek tells you that the number of two-point shots that he made is five more than the number of three-point shots. How many combinations can you come up with that fit this scenario? (Don't worry about the total number of points.)

Number of Two-Pointers	Number of Three-Pointers

Write an equation to describe the data.

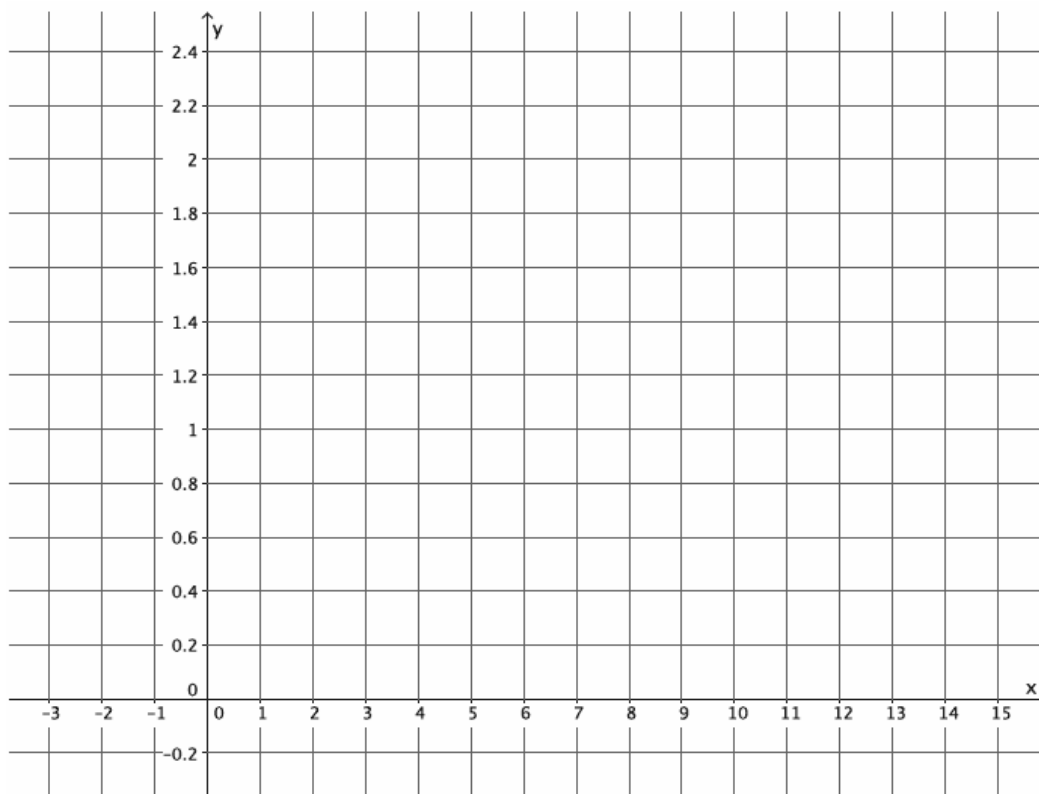
3. Which pair of numbers from your table in Exercise 2 would show Derek's actual score of 30 points?
4. Efrain and Fernie are on a road trip. Each of them drives at a constant speed. Efrain is a safe driver and travels 45 miles per hour for the entire trip. Fernie is not such a safe driver. He drives 70 miles per hour throughout the trip. Fernie and Efrain left from the same location, but Efrain left at 8:00 a.m., and Fernie left at 11:00 a.m. Assuming they take the same route, will Fernie ever catch up to Efrain? If so, approximately when?
- Write the linear equation that represents Efrain's constant speed. Make sure to include in your equation the extra time that Efrain was able to travel.
  - Write the linear equation that represents Fernie's constant speed.

- c. Write the system of linear equations that represents this situation.
- d. Sketch the graphs of the two linear equations.



- e. Will Fernie ever catch up to Efrain? If so, approximately when?
- f. At approximately what point do the graphs of the lines intersect?

5. Jessica and Karl run at constant speeds. Jessica can run 3 miles in 24 minutes. Karl can run 2 miles in 14 minutes. They decide to race each other. As soon as the race begins, Karl trips and takes 2 minutes to recover.
- Write the linear equation that represents Jessica's constant speed. Make sure to include in your equation the extra time that Jessica was able to run.
  - Write the linear equation that represents Karl's constant speed.
  - Write the system of linear equations that represents this situation.
  - Sketch the graphs of the two linear equations.





### Lesson Summary

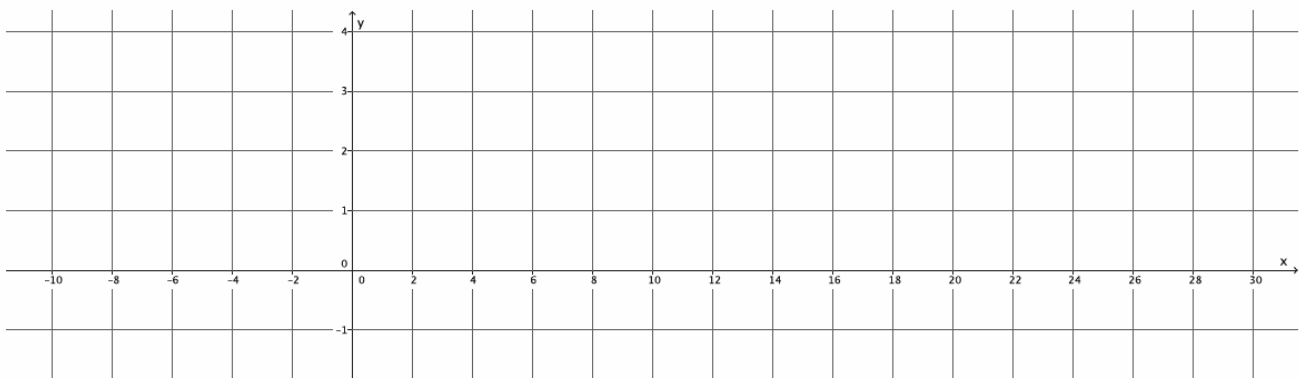
A *system of linear equations* is a set of two or more linear equations. When graphing a pair of linear equations in two variables, both equations in the system are graphed on the same coordinate plane.

A *solution to a system of two linear equations in two variables* is an ordered pair of numbers that is a solution to both equations. For example, the solution to the system of linear equations  $\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$  is the ordered pair (5, 1) because substituting 5 in for  $x$  and 1 in for  $y$  results in two true equations:  $5 + 1 = 6$  and  $5 - 1 = 4$ .

Systems of linear equations are notated using brackets to group the equations, for example:  $\begin{cases} y = \frac{1}{8}x + \frac{5}{2} \\ y = \frac{4}{25}x \end{cases}$ .

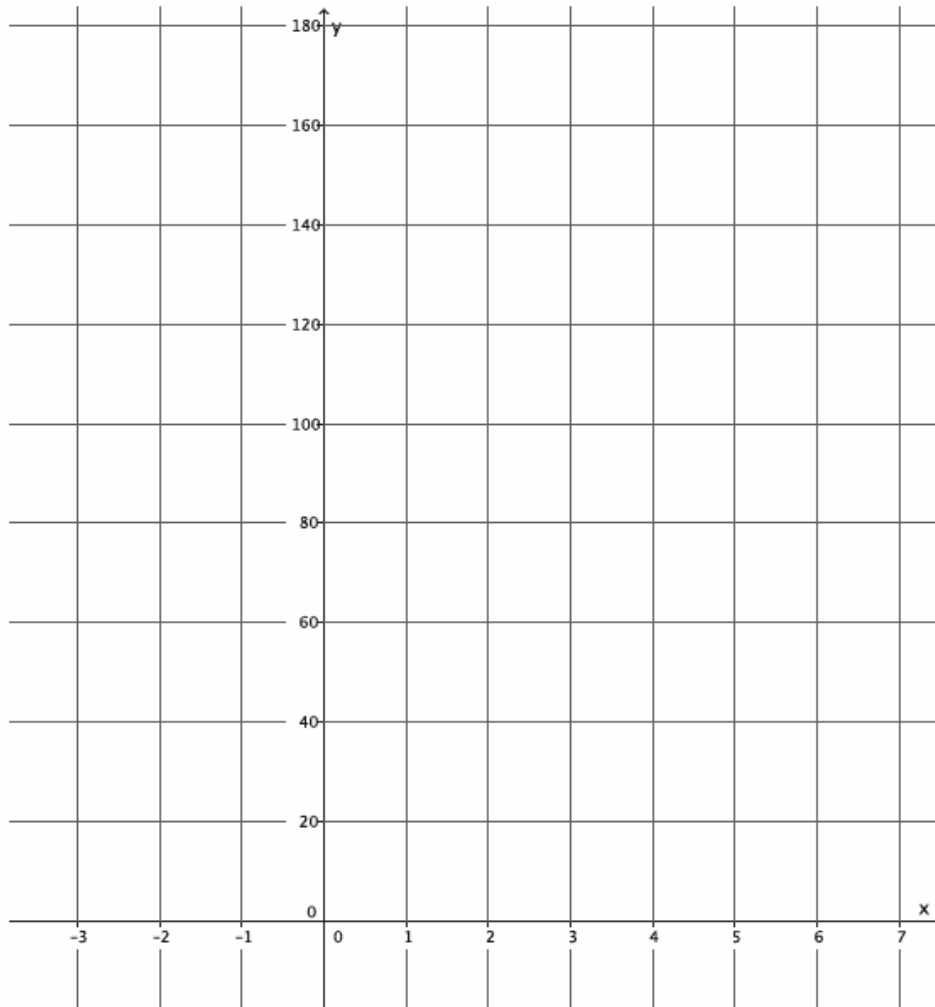
### Problem Set

- Jeremy and Gerardo run at constant speeds. Jeremy can run 1 mile in 8 minutes, and Gerardo can run 3 miles in 33 minutes. Jeremy started running 10 minutes after Gerardo. Assuming they run the same path, when will Jeremy catch up to Gerardo?
  - Write the linear equation that represents Jeremy's constant speed.
  - Write the linear equation that represents Gerardo's constant speed. Make sure to include in your equation the extra time that Gerardo was able to run.
  - Write the system of linear equations that represents this situation.
  - Sketch the graphs of the two equations.



- Will Jeremy ever catch up to Gerardo? If so, approximately when?
- At approximately what point do the graphs of the lines intersect?

2. Two cars drive from town A to town B at constant speeds. The blue car travels 25 miles per hour, and the red car travels 60 miles per hour. The blue car leaves at 9:30 a.m., and the red car leaves at noon. The distance between the two towns is 150 miles.
- a. Who will get there first? Write and graph the system of linear equations that represents this situation.



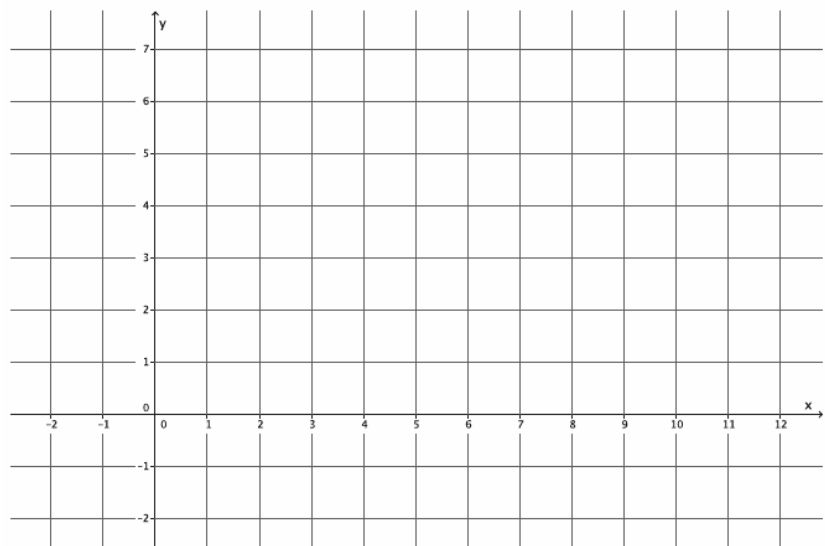
- b. At approximately what point do the graphs of the lines intersect?

## Lesson 25: Geometric Interpretation of the Solutions of a Linear System

### Classwork

#### Exploratory Challenge/Exercises 1–5

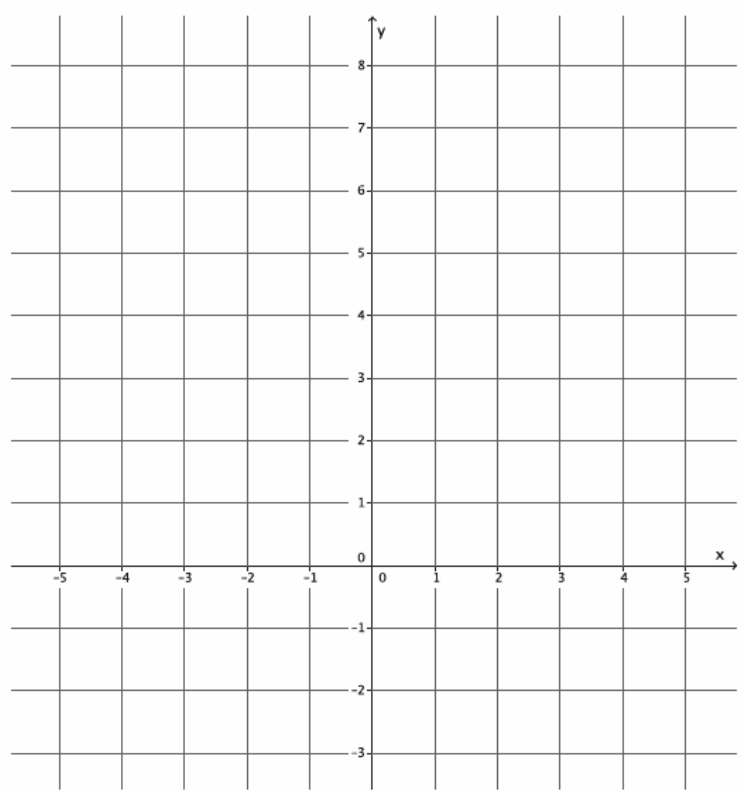
1. Sketch the graphs of the linear system on a coordinate plane: 
$$\begin{cases} 2y + x = 12 \\ y = \frac{5}{6}x - 2 \end{cases}$$



- Name the ordered pair where the graphs of the two linear equations intersect.
- Verify that the ordered pair named in part (a) is a solution to  $2y + x = 12$ .
- Verify that the ordered pair named in part (a) is a solution to  $y = \frac{5}{6}x - 2$ .

- d. Could the point  $(4, 4)$  be a solution to the system of linear equations? That is, would  $(4, 4)$  make both equations true? Why or why not?

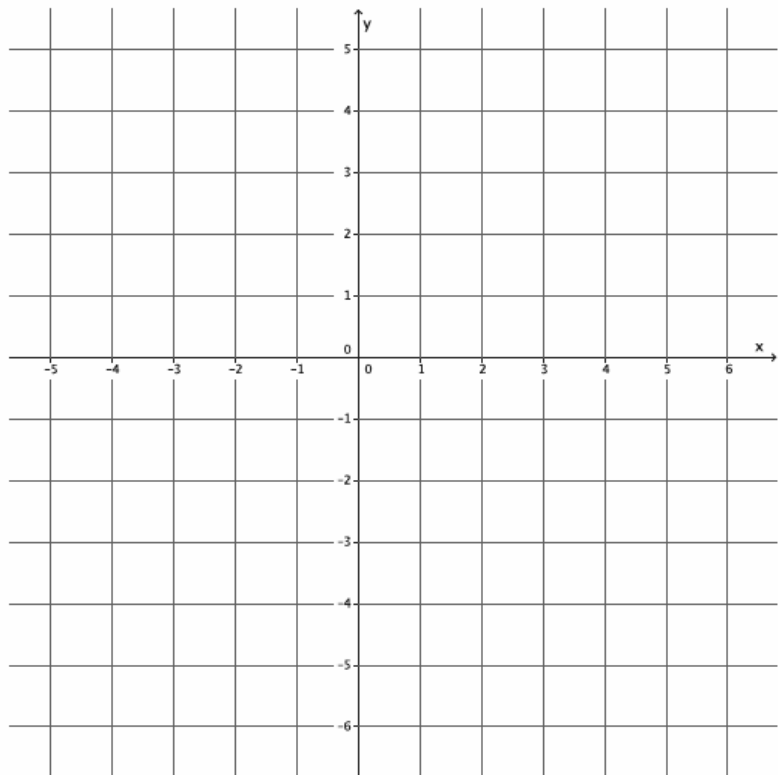
2. Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} x + y = -2 \\ y = 4x + 3 \end{cases}$ .



- a. Name the ordered pair where the graphs of the two linear equations intersect.
- b. Verify that the ordered pair named in part (a) is a solution to  $x + y = -2$ .

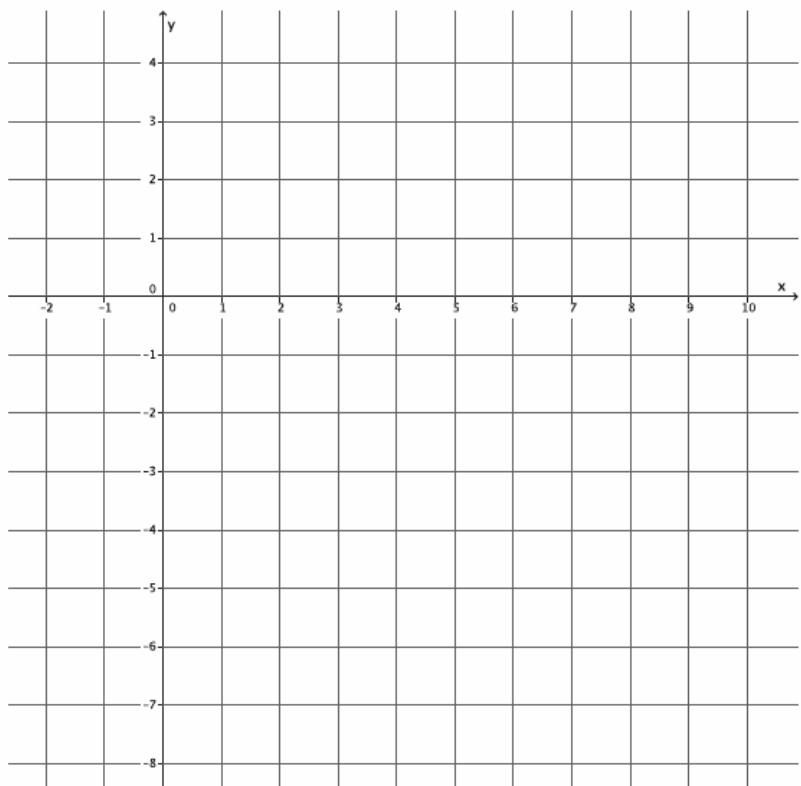
- c. Verify that the ordered pair named in part (a) is a solution to  $y = 4x + 3$ .
- d. Could the point  $(-4, 2)$  be a solution to the system of linear equations? That is, would  $(-4, 2)$  make both equations true? Why or why not?

3. Sketch the graphs of the linear system on a coordinate plane: 
$$\begin{cases} 3x + y = -3 \\ -2x + y = 2 \end{cases}$$

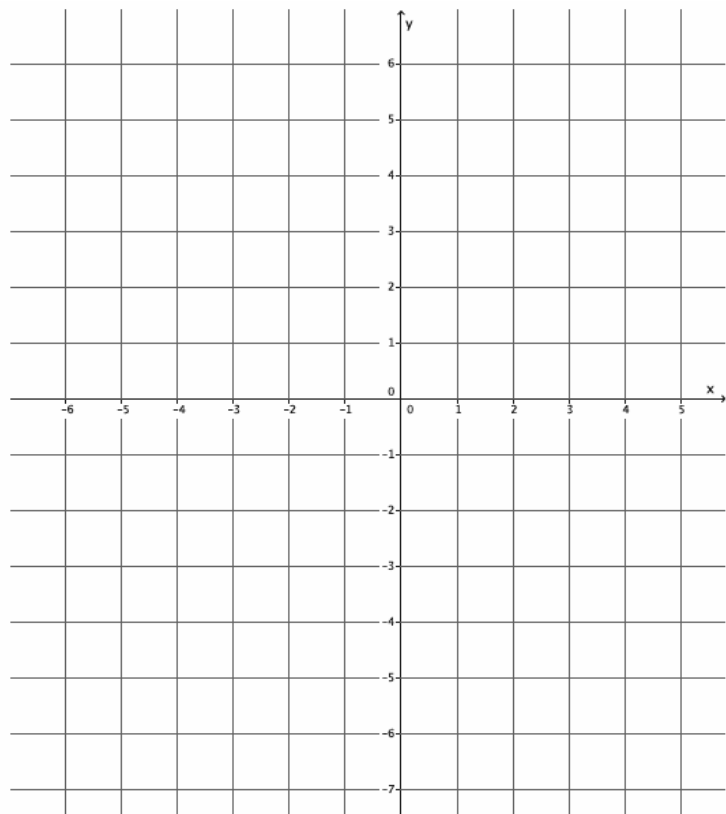


- a. Name the ordered pair where the graphs of the two linear equations intersect.

- b. Verify that the ordered pair named in part (a) is a solution to  $3x + y = -3$ .
- c. Verify that the ordered pair named in part (a) is a solution to  $-2x + y = 2$ .
- d. Could the point  $(1, 4)$  be a solution to the system of linear equations? That is, would  $(1, 4)$  make both equations true? Why or why not?
4. Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} 2x - 3y = 18 \\ 2x + y = 2 \end{cases}$ .



- a. Name the ordered pair where the graphs of the two linear equations intersect.
- b. Verify that the ordered pair named in part (a) is a solution to  $2x - 3y = 18$ .
- c. Verify that the ordered pair named in part (a) is a solution to  $2x + y = 2$ .
- d. Could the point  $(3, -1)$  be a solution to the system of linear equations? That is, would  $(3, -1)$  make both equations true? Why or why not?
5. Sketch the graphs of the linear system on a coordinate plane: 
$$\begin{cases} y - x = 3 \\ y = -4x - 2 \end{cases}$$



- a. Name the ordered pair where the graphs of the two linear equations intersect.
- b. Verify that the ordered pair named in part (a) is a solution to  $y - x = 3$ .
- c. Verify that the ordered pair named in part (a) is a solution to  $y = -4x - 2$ .
- d. Could the point  $(-2, 6)$  be a solution to the system of linear equations? That is, would  $(-2, 6)$  make both equations true? Why or why not?

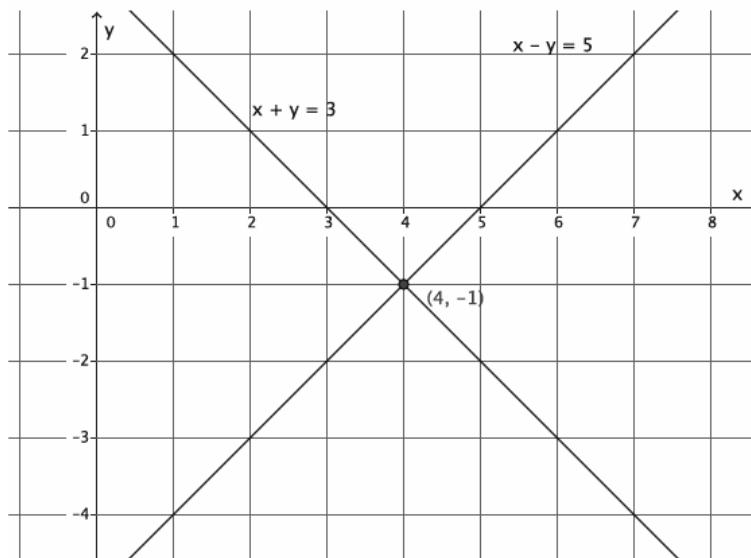
**Exercise 6**

6. Write two different systems of equations with  $(1, -2)$  as the solution.

**Lesson Summary**

When the graphs of a system of linear equations are sketched, and if they are not parallel lines, then the point of intersection of the lines of the graph represents the solution to the system. Two distinct lines intersect at most at one point, if they intersect. The coordinates of that point  $(x, y)$  represent values that make both equations of the system true.

Example: The system  $\begin{cases} x + y = 3 \\ x - y = 5 \end{cases}$  graphs as shown below.



The lines intersect at  $(4, -1)$ . That means the equations in the system are true when  $x = 4$  and  $y = -1$ .

$$\begin{aligned} x + y &= 3 \\ 4 + (-1) &= 3 \\ 3 &= 3 \end{aligned}$$

$$\begin{aligned} x - y &= 5 \\ 4 - (-1) &= 5 \\ 5 &= 5 \end{aligned}$$

## Problem Set

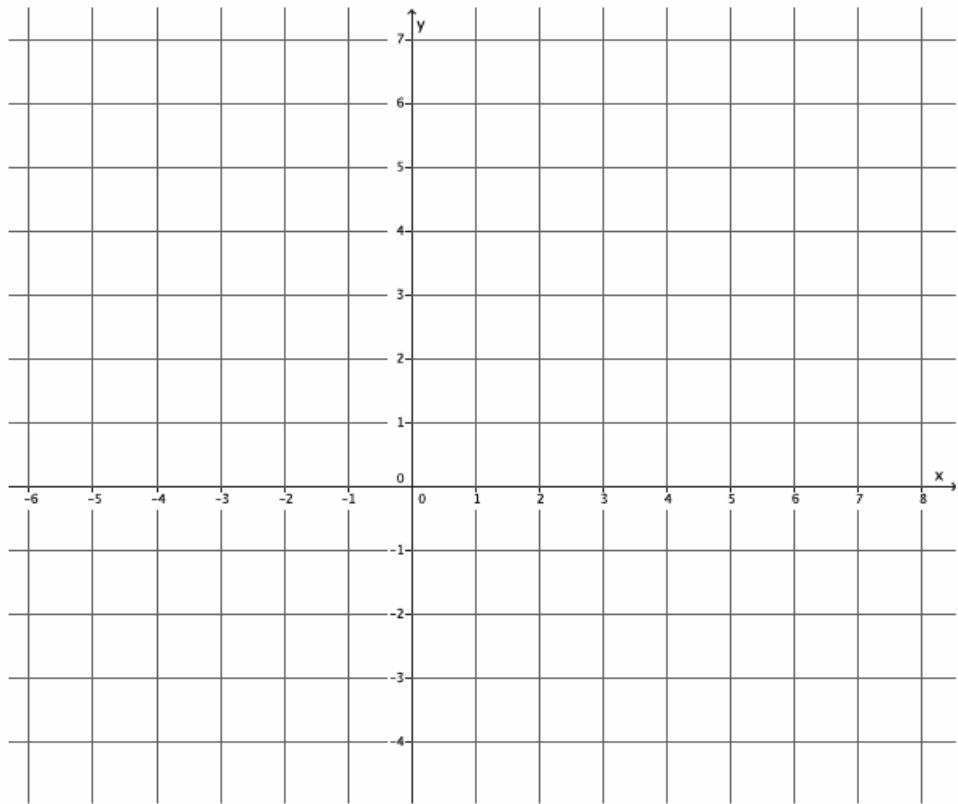
- Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} y = \frac{1}{3}x + 1 \\ y = -3x + 11 \end{cases}$ .
  - Name the ordered pair where the graphs of the two linear equations intersect.
  - Verify that the ordered pair named in part (a) is a solution to  $y = \frac{1}{3}x + 1$ .
  - Verify that the ordered pair named in part (a) is a solution to  $y = -3x + 11$ .
- Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} y = \frac{1}{2}x + 4 \\ x + 4y = 4 \end{cases}$ .
  - Name the ordered pair where the graphs of the two linear equations intersect.
  - Verify that the ordered pair named in part (a) is a solution to  $y = \frac{1}{2}x + 4$ .
  - Verify that the ordered pair named in part (a) is a solution to  $x + 4y = 4$ .
- Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} y = 2 \\ x + 2y = 10 \end{cases}$ .
  - Name the ordered pair where the graphs of the two linear equations intersect.
  - Verify that the ordered pair named in part (a) is a solution to  $y = 2$ .
  - Verify that the ordered pair named in part (a) is a solution to  $x + 2y = 10$ .
- Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} -2x + 3y = 18 \\ 2x + 3y = 6 \end{cases}$ .
  - Name the ordered pair where the graphs of the two linear equations intersect.
  - Verify that the ordered pair named in part (a) is a solution to  $-2x + 3y = 18$ .
  - Verify that the ordered pair named in part (a) is a solution to  $2x + 3y = 6$ .
- Sketch the graphs of the linear system on a coordinate plane:  $\begin{cases} x + 2y = 2 \\ y = \frac{2}{3}x - 6 \end{cases}$ .
  - Name the ordered pair where the graphs of the two linear equations intersect.
  - Verify that the ordered pair named in part (a) is a solution to  $x + 2y = 2$ .
  - Verify that the ordered pair named in part (a) is a solution to  $y = \frac{2}{3}x - 6$ .
- Without sketching the graph, name the ordered pair where the graphs of the two linear equations intersect.
$$\begin{cases} x = 2 \\ y = -3 \end{cases}$$

## Lesson 26: Characterization of Parallel Lines

### Classwork

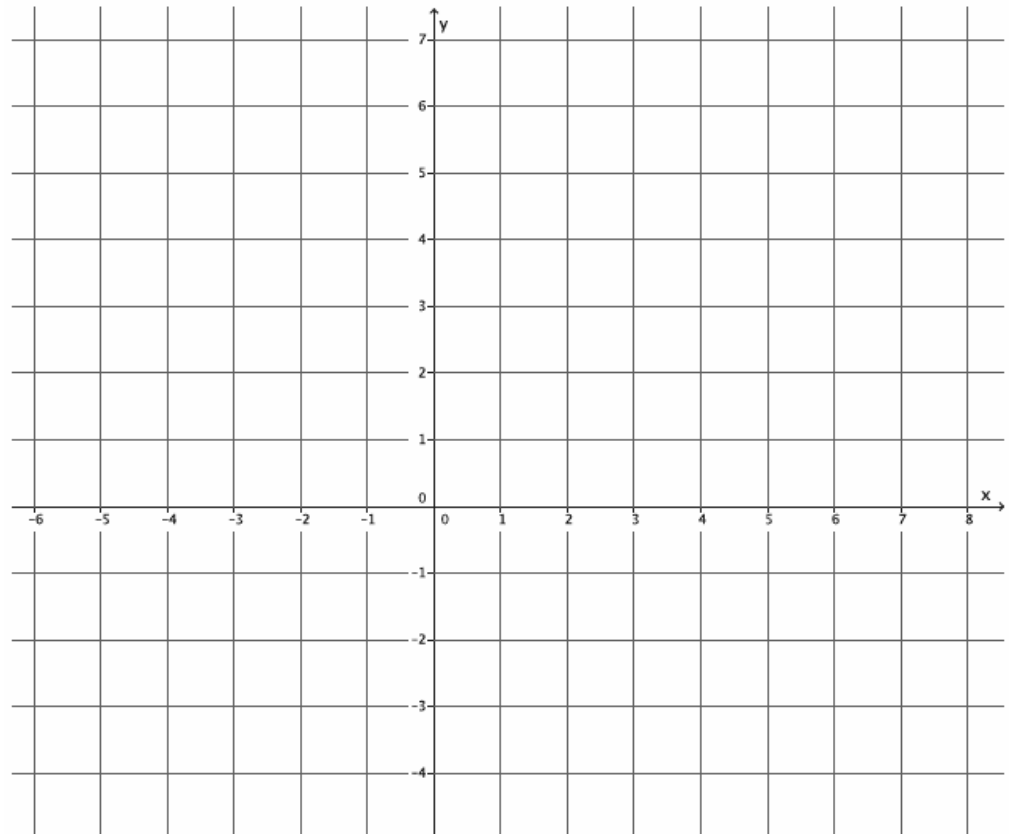
#### Exercises

1. Sketch the graphs of the system. 
$$\begin{cases} y = \frac{2}{3}x + 4 \\ y = \frac{4}{6}x - 3 \end{cases}$$



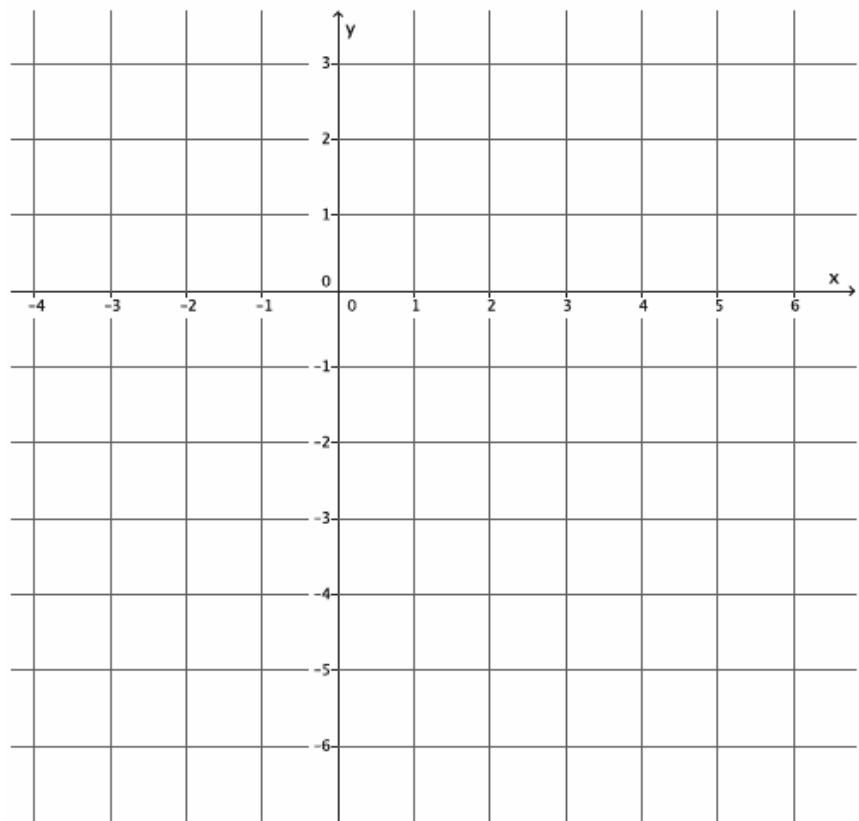
- a. Identify the slope of each equation. What do you notice?
- b. Identify the y-intercept point of each equation. Are the y-intercept points the same or different?

2. Sketch the graphs of the system. 
$$\begin{cases} y = -\frac{5}{4}x + 7 \\ y = -\frac{5}{4}x + 2 \end{cases}$$



- a. Identify the slope of each equation. What do you notice?
- b. Identify the  $y$ -intercept point of each equation. Are the  $y$ -intercept points the same or different?

3. Sketch the graphs of the system.  $\begin{cases} y = 2x - 5 \\ y = 2x - 1 \end{cases}$



- a. Identify the slope of each equation. What do you notice?
- b. Identify the  $y$ -intercept point of each equation. Are the  $y$ -intercept points the same or different?

4. Write a system of equations that has no solution.

5. Write a system of equations that has  $(2, 1)$  as a solution.

6. How can you tell if a system of equations has a solution or not?

7. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} 6x - 2y = 5 \\ 4x - 3y = 5 \end{cases}$$

8. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} -2x + 8y = 14 \\ x = 4y + 1 \end{cases}$$

9. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} 12x + 3y = -2 \\ 4x + y = 7 \end{cases}$$

10. Genny babysits for two different families. One family pays her \$6 each hour and a bonus of \$20 at the end of the night. The other family pays her \$3 every half hour and a bonus of \$25 at the end of the night. Write and solve the system of equations that represents this situation. At what number of hours do the two families pay the same for babysitting services from Genny?

**Lesson Summary**

By definition, parallel lines do not intersect; therefore, a system of linear equations whose graphs are parallel lines will have no solution.

Parallel lines have the same slope but no common point. One can verify that two lines are parallel by comparing their slopes and their  $y$ -intercept points.

**Problem Set**

Answer Problems 1–5 without graphing the equations.

1. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} 2x + 5y = 9 \\ -4x - 10y = 4 \end{cases}$$

2. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} \frac{3}{4}x - 3 = y \\ 4x - 3y = 5 \end{cases}$$

3. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} x + 7y = 8 \\ 7x - y = -2 \end{cases}$$

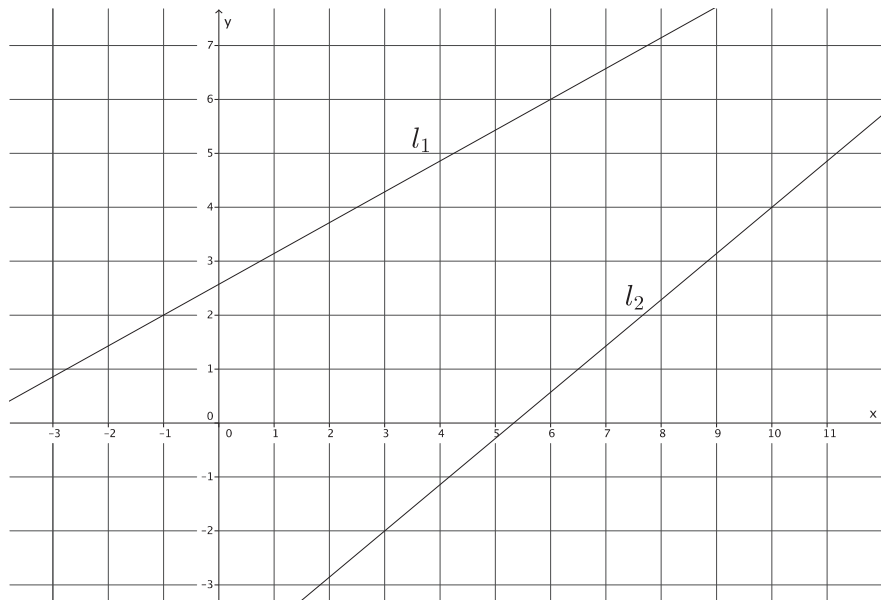
4. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} y = 5x + 12 \\ 10x - 2y = 1 \end{cases}$$

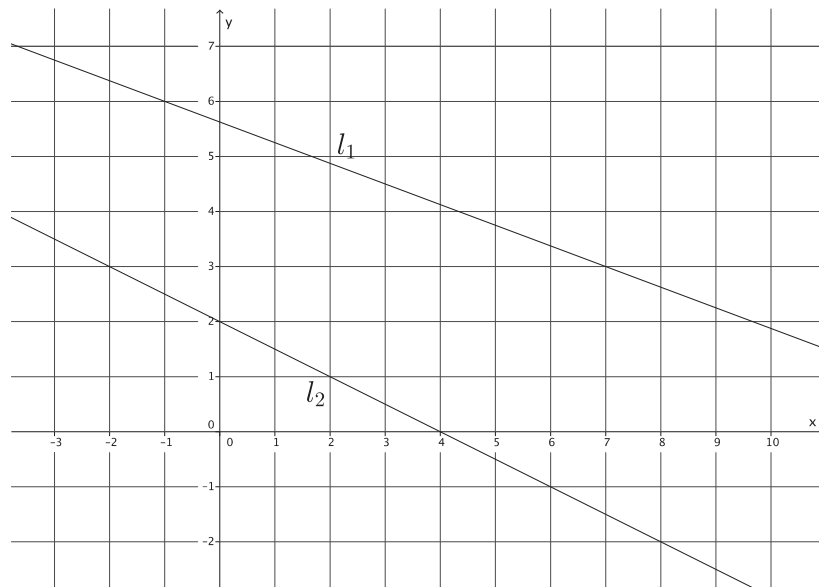
5. Does the system of linear equations shown below have a solution? Explain.

$$\begin{cases} y = \frac{5}{3}x + 15 \\ 5x - 3y = 6 \end{cases}$$

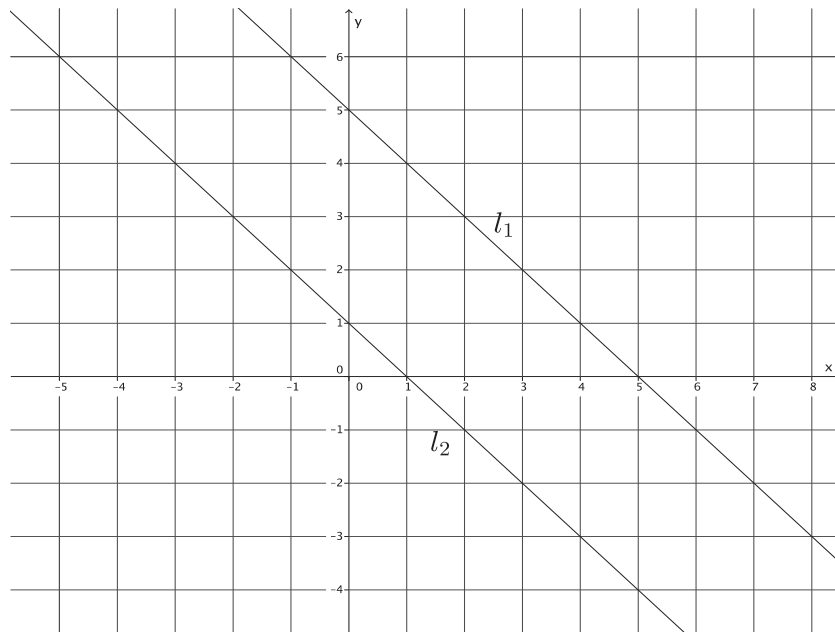
6. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



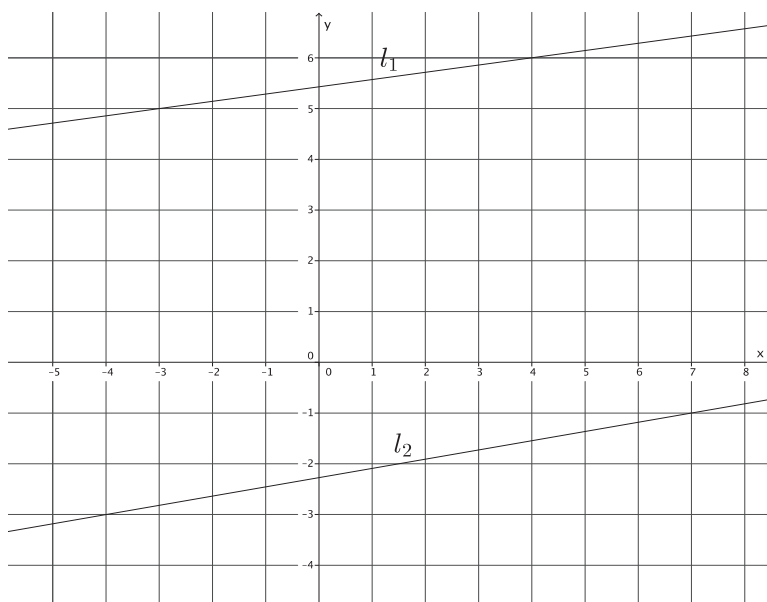
7. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



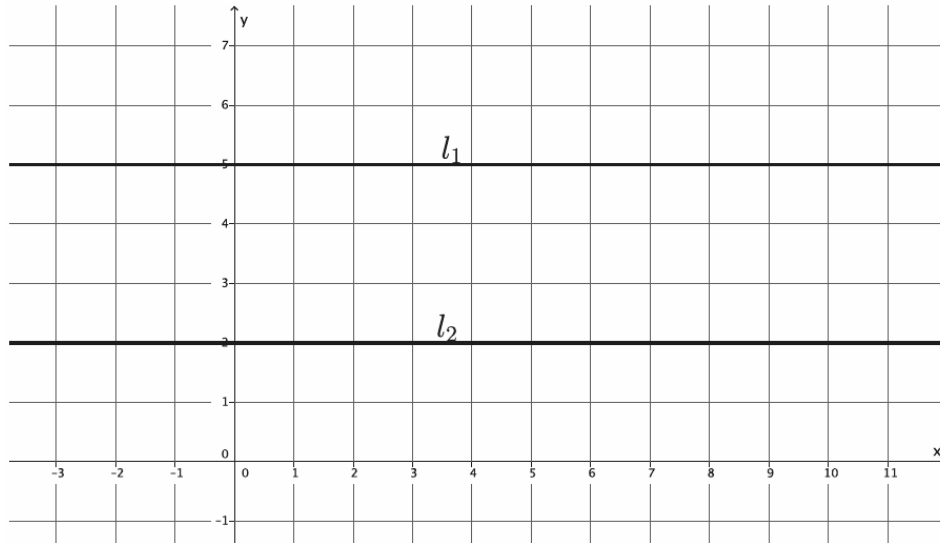
8. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



9. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



10. Given the graphs of a system of linear equations below, is there a solution to the system that we cannot see on this portion of the coordinate plane? That is, will the lines intersect somewhere on the plane not represented in the picture? Explain.



## Lesson 27: Nature of Solutions of a System of Linear Equations

### Classwork

#### Exercises

Determine the nature of the solution to each system of linear equations.

1. 
$$\begin{cases} 3x + 4y = 5 \\ y = -\frac{3}{4}x + 1 \end{cases}$$

2. 
$$\begin{cases} 7x + 2y = -4 \\ x - y = 5 \end{cases}$$

3. 
$$\begin{cases} 9x + 6y = 3 \\ 3x + 2y = 1 \end{cases}$$

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

4. 
$$\begin{cases} 3x + 3y = -21 \\ x + y = -7 \end{cases}$$

5. 
$$\begin{cases} y = \frac{3}{2}x - 1 \\ 3y = x + 2 \end{cases}$$

6. 
$$\begin{cases} x = 12y - 4 \\ x = 9y + 7 \end{cases}$$

7. Write a system of equations with  $(4, -5)$  as its solution.

**Lesson Summary**

A system of linear equations can have a unique solution, no solution, or infinitely many solutions.

Systems with a unique solution are comprised of two linear equations whose graphs have different slopes; that is, their graphs in a coordinate plane will be two distinct lines that intersect at only one point.

Systems with no solutions are comprised of two linear equations whose graphs have the same slope but different  $y$ -intercept points; that is, their graphs in a coordinate plane will be two parallel lines (with no intersection).

Systems with infinitely many solutions are comprised of two linear equations whose graphs have the same slope and the same  $y$ -intercept point; that is, their graphs in a coordinate plane are the same line (i.e., every solution to one equation will be a solution to the other equation).

A system of linear equations can be solved using a substitution method. That is, if two expressions are equal to the same value, then they can be written equal to one another.

Example:

$$\begin{cases} y = 5x - 8 \\ y = 6x + 3 \end{cases}$$

Since both equations in the system are equal to  $y$ , we can write the equation  $5x - 8 = 6x + 3$  and use it to solve for  $x$  and then the system.

Example:

$$\begin{cases} 3x = 4y + 2 \\ x = y + 5 \end{cases}$$

Multiply each term of the equation  $x = y + 5$  by 3 to produce the equivalent equation  $3x = 3y + 15$ . As in the previous example, since both equations equal  $3x$ , we can write  $4y + 2 = 3y + 15$ . This equation can be used to solve for  $y$  and then the system.

**Problem Set**

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1. 
$$\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$$

2. 
$$\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$$

3. 
$$\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$$

4. 
$$\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$$

5. 
$$\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$$

6. 
$$\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$$

7. 
$$\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$$

8. 
$$\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$$

9. 
$$\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$$

10. 
$$\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$$

11. Write a system of linear equations with  $(-3, 9)$  as its solution.

## Lesson 28: Another Computational Method of Solving a Linear System

### Classwork

#### Example 1

Use what you noticed about adding equivalent expressions to solve the following system by elimination:

$$\begin{cases} 6x - 5y = 21 \\ 2x + 5y = -5 \end{cases}$$

#### Example 2

Solve the following system by elimination:

$$\begin{cases} -2x + 7y = 5 \\ 4x - 2y = 14 \end{cases}$$

**Example 3**

Solve the following system by elimination:

$$\begin{cases} 7x - 5y = -2 \\ 3x - 3y = 7 \end{cases}$$

**Exercises**

Each of the following systems has a solution. Determine the solution to the system by eliminating one of the variables. Verify the solution using the graph of the system.

1. 
$$\begin{cases} 6x - 7y = -10 \\ 3x + 7y = -8 \end{cases}$$

$$2. \begin{cases} x - 4y = 7 \\ 5x + 9y = 6 \end{cases}$$

$$3. \begin{cases} 2x - 3y = -5 \\ 3x + 5y = 1 \end{cases}$$

### Lesson Summary

Systems of linear equations can be solved by eliminating one of the variables from the system. One way to eliminate a variable is by setting both equations equal to the same variable and then writing the expressions equal to one another.

Example: Solve the system  $\begin{cases} y = 3x - 4 \\ y = 2x + 1 \end{cases}$

Since the expressions  $3x - 4$  and  $2x + 1$  are both equal to  $y$ , they can be set equal to each other and the new equation can be solved for  $x$ :

$$3x - 4 = 2x + 1$$

Another way to eliminate a variable is by multiplying each term of an equation by the same constant to make an equivalent equation. Then, use the equivalent equation to eliminate one of the variables and solve the system.

Example: Solve the system  $\begin{cases} 2x + y = 8 \\ x + y = 10 \end{cases}$

Multiply the second equation by  $-2$  to eliminate the  $x$ .

$$\begin{aligned} -2(x + y) &= -2(10) \\ -2x - 2y &= -20 \end{aligned}$$

Now we have the system  $\begin{cases} 2x + y = 8 \\ -2x - 2y = -20 \end{cases}$

When the equations are added together, the  $x$  is eliminated.

$$\begin{aligned} 2x + y - 2x - 2y &= 8 + (-20) \\ y - 2y &= 8 + (-20) \end{aligned}$$

Once a solution has been found, verify the solution graphically or by substitution.

### Problem Set

Determine the solution, if it exists, for each system of linear equations. Verify your solution on the coordinate plane.

1.  $\begin{cases} \frac{1}{2}x + 5 = y \\ 2x + y = 1 \end{cases}$

2.  $\begin{cases} 9x + 2y = 9 \\ -3x + y = 2 \end{cases}$

3.  $\begin{cases} y = 2x - 2 \\ 2y = 4x - 4 \end{cases}$

4. 
$$\begin{cases} 8x + 5y = 19 \\ -8x + y = -1 \end{cases}$$

5. 
$$\begin{cases} x + 3 = y \\ 3x + 4y = 7 \end{cases}$$

6. 
$$\begin{cases} y = 3x + 2 \\ 4y = 12 + 12x \end{cases}$$

7. 
$$\begin{cases} 4x - 3y = 16 \\ -2x + 4y = -2 \end{cases}$$

8. 
$$\begin{cases} 2x + 2y = 4 \\ 12 - 3x = 3y \end{cases}$$

9. 
$$\begin{cases} y = -2x + 6 \\ 3y = x - 3 \end{cases}$$

10. 
$$\begin{cases} y = 5x - 1 \\ 10x = 2y + 2 \end{cases}$$

11. 
$$\begin{cases} 3x - 5y = 17 \\ 6x + 5y = 10 \end{cases}$$

12. 
$$\begin{cases} y = \frac{4}{3}x - 9 \\ y = x + 3 \end{cases}$$

13. 
$$\begin{cases} 4x - 7y = 11 \\ x + 2y = 10 \end{cases}$$

14. 
$$\begin{cases} 21x + 14y = 7 \\ 12x + 8y = 16 \end{cases}$$

## Lesson 29: Word Problems

### Classwork

#### Example 1

The sum of two numbers is 361, and the difference between the two numbers is 173. What are the two numbers?

#### Example 2

There are 356 eighth-grade students at Euclid's Middle School. Thirty-four more than four times the number of girls is equal to half the number of boys. How many boys are in eighth grade at Euclid's Middle School? How many girls?

**Example 3**

A family member has some five-dollar bills and one-dollar bills in her wallet. Altogether she has 18 bills and a total of \$62. How many of each bill does she have?

**Example 4**

A friend bought 2 boxes of pencils and 8 notebooks for school, and it cost him \$11. He went back to the store the same day to buy school supplies for his younger brother. He spent \$11.25 on 3 boxes of pencils and 5 notebooks. How much would 7 notebooks cost?



3. The sum of the measures of angles  $x$  and  $y$  is  $127^\circ$ . If the measure of  $\angle x$  is  $34^\circ$  more than half the measure of  $\angle y$ , what is the measure of each angle?

**Problem Set**

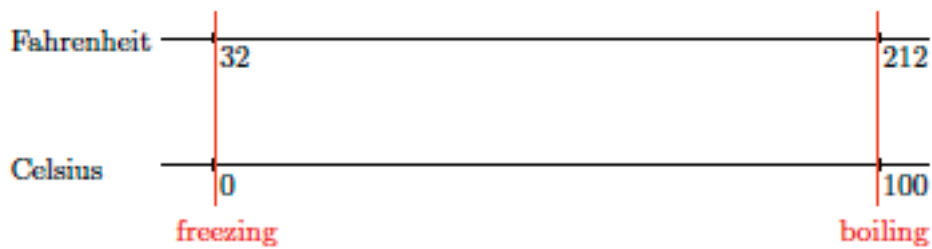
1. Two numbers have a sum of 1,212 and a difference of 518. What are the two numbers?
2. The sum of the ages of two brothers is 46. The younger brother is 10 more than a third of the older brother's age. How old is the younger brother?
3. One angle measures 54 more degrees than 3 times another angle. The angles are supplementary. What are their measures?
4. Some friends went to the local movie theater and bought four large buckets of popcorn and six boxes of candy. The total for the snacks was \$46.50. The last time you were at the theater, you bought a large bucket of popcorn and a box of candy, and the total was \$9.75. How much would 2 large buckets of popcorn and 3 boxes of candy cost?
5. You have 59 total coins for a total of \$12.05. You only have quarters and dimes. How many of each coin do you have?
6. A piece of string is 112 inches long. Isabel wants to cut it into 2 pieces so that one piece is three times as long as the other. How long is each piece?

## Lesson 30: Conversion Between Celsius and Fahrenheit

### Classwork

#### Mathematical Modeling Exercise

- (1) If  $t$  is a number, what is the degree in Fahrenheit that corresponds to  $t^{\circ}\text{C}$ ?
- (2) If  $t$  is a number, what is the degree in Fahrenheit that corresponds to  $(-t)^{\circ}\text{C}$ ?





**Problem Set**

1. Does the equation  $t^{\circ}\text{C} = (32 + 1.8t)^{\circ}\text{F}$  work for any rational number  $t$ ? Check that it does with  $t = 8\frac{2}{3}$  and  $t = -8\frac{2}{3}$ .
2. Knowing that  $t^{\circ}\text{C} = \left(32 + \frac{9}{5}t\right)^{\circ}\text{F}$  for any rational  $t$ , show that for any rational number  $d$ ,  $d^{\circ}\text{F} = \left(\frac{5}{9}(d - 32)\right)^{\circ}\text{C}$ .
3. Drake was trying to write an equation to help him predict the cost of his monthly phone bill. He is charged \$35 just for having a phone, and his only additional expense comes from the number of texts that he sends. He is charged \$0.05 for each text. Help Drake out by completing parts (a)–(f).
  - a. How much was his phone bill in July when he sent 750 texts?
  - b. How much was his phone bill in August when he sent 823 texts?
  - c. How much was his phone bill in September when he sent 579 texts?
  - d. Let  $y$  represent the total cost of Drake's phone bill. Write an equation that represents the total cost of his phone bill in October if he sends  $t$  texts.
  - e. Another phone plan charges \$20 for having a phone and \$0.10 per text. Let  $y$  represent the total cost of the phone bill for sending  $t$  texts. Write an equation to represent his total bill.
  - f. Write your equations in parts (d) and (e) as a system of linear equations, and solve. Interpret the meaning of the solution in terms of the phone bill.

## Lesson 31: System of Equations Leading to Pythagorean Triples

### Classwork

#### Exercises

1. Identify two Pythagorean triples using the known triple 3, 4, 5 (other than 6, 8, 10).
2. Identify two Pythagorean triples using the known triple 5, 12, 13.
3. Identify two triples using either 3, 4, 5 or 5, 12, 13.

Use the system  $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$  to find Pythagorean triples for the given values of  $s$  and  $t$ . Recall that the solution in the form of  $(\frac{c}{b}, \frac{a}{b})$  is the triple  $a, b, c$ .

4.  $s = 4, t = 5$

5.  $s = 7, t = 10$

6.  $s = 1, t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Exercises 4–6. Show your work below.

**Lesson Summary**

A Pythagorean triple is a set of three positive integers that satisfies the equation  $a^2 + b^2 = c^2$ .

An infinite number of Pythagorean triples can be found by multiplying the numbers of a known triple by a whole number. For example, 3, 4, 5 is a Pythagorean triple. Multiply each number by 7, and then you have 21, 28, 35, which is also a Pythagorean triple.

The system of linear equations,  $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$  can be used to find Pythagorean triples, just like the Babylonians did 4,000 years ago.

**Problem Set**

1. Explain in terms of similar triangles why it is that when you multiply the known Pythagorean triple 3, 4, 5 by 12, it generates a Pythagorean triple.
2. Identify three Pythagorean triples using the known triple 8, 15, 17.
3. Identify three triples (numbers that satisfy  $a^2 + b^2 = c^2$ , but  $a, b, c$  are not whole numbers) using the triple 8, 15, 17.

Use the system  $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$  to find Pythagorean triples for the given values of  $s$  and  $t$ . Recall that the solution, in the form of  $(\frac{c}{b}, \frac{a}{b})$ , is the triple  $a, b, c$ .

4.  $s = 2, t = 9$
5.  $s = 6, t = 7$
6.  $s = 3, t = 4$
7. Use a calculator to verify that you found a Pythagorean triple in each of the Problems 4–6. Show your work.